Elementary Matrices and the LU-decomposition:

1. Write the following matrices and their inverses as a product of elementary matrices:

$$\begin{bmatrix} 1 & 4 \\ 2 & 7 \end{bmatrix}, \quad \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \\ 1 & 1 & 0 \end{bmatrix}$$

2. Express the matrix

$$\begin{bmatrix} 0 & 1 & 7 & 8 \\ 1 & 3 & 3 & 8 \\ -2 & -5 & 1 & -8 \end{bmatrix}$$

as a product $A = EFGR$ where E, F and G are elementary matrices, and R is in row-echelon form.

3. Find the LU-decomposition of the matrix

$$A = \begin{bmatrix} -3 & 12 & -6 \\ 1 & -2 & 2 \\ 0 & 1 & 1 \end{bmatrix}$$

and use it to solve the system

$$A \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} -33 \\ 7 \\ -1 \end{bmatrix}$$