METU

Northern Cyprus Campus

| MAT 219 | Introduction to Differential Equations | Short Exam III | 10.05.2015 |
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| Last Name: <br> Name: Key <br> Student No: | Dept./Sec.: <br> Time : 12: 40 <br> Duration : 40 *minutes* | Signature | |
| 2 QUESTIONS | | TOTAL 10 POINTS | |
| 1 | 2 | | |

**Q1 (5 pts.)** Using the Method of Undetermined Coefficients, write down the general solution to the differential equation $y^{(3)} - y' = te^{-t} + 2\sin(t)$. **DO NOT EVALUATE THE COEFFICIENTS**.

$$y^{(3)} - y' = te^{-t} + 2\sin(t)$$

$$t^3 - t = 0$$

$$t_1 = 0, \quad t_2 = 1, \quad t_3 = -1$$

$$y_h = c_1 + c_2 e^{t} + c_3 e^{-t}$$

$$Y_1(t) = (A_0 t + A_1)e^{-t} \cdot t$$

$$Y_2(t) = B\cos t + C\sin t$$

$$y_c = c_1 + c_2 e^{t} + c_3 e^{-t} + (A_0 t + A_1)e^{-t} \cdot t +$$

$$+ B\cos t + C\sin t$$

**Q2 (5 pts.)** Find the general solution to the differential equation $ty'' - y' + 4t^3 y = 0$, $t > 0$ with the given first solution $y_1(t) = \cos(t^2)$.

**Bonus (2 pts)** Verify Abel's formula (or theorem) by computing the Wronskian of the solutions.

$$ty'' - y' + 4t^3 y = 0, \quad t>0, \quad y_1(t) = \cos(t^2)$$

$$y_2 = v\cos(t^2)$$

$$y' = v'\cos(t^2) - 2tv\sin(t^2)$$

$$y'' = v''\cos(t^2) - 4tv'\sin(t^2) - 2v\sin(t^2) - 4t^2 v\cos(t^2)$$

$$tv''\cos(t^2) - 4t^2v'\sin(t^2) - \cancel{2vt\sin(t^2)} - \cancel{4t^3 v\cos(t^2)}$$
$$- v'\cos(t^2) + \cancel{2tv\sin(t^2)} + \cancel{4t^3 v\cos t^2} = 0$$

$$\frac{dv'}{v'} = 4t\tan(t^2) + \frac{1}{t})dt$$

$$\ln|v'| = -2\ln|\cos(t^2)| + \ln t + c$$

$$v' = c\,t\cos^{-2}(t^2)$$

$$v = c\int \frac{t\,dt}{\cos^2(t^2)} + K$$

$$v = \frac{1}{2}c\tan(t^2) + K.$$

$$\boxed{W = ce^{-\int p_1(t)dt} = = c\cdot e^{\int \frac{1}{t}dt} = = ce^{\ln t} = ct.}$$

$K=0$

$\frac{c}{2} = 1$, $\quad v = \tan(t^2)$

$$y_2(t) = \tan(t^2)\cdot\cos(t^2) = \sin(t^2)$$

$$y_c(t) = c_1\cos(t^2) + c_2\sin(t^2)$$

$$W = \begin{vmatrix} \cos(t^2) & \sin(t^2) \\ -2t\sin(t^2) & 2t\cos(t^2) \end{vmatrix} = 2t\cos^2(t^2) + 2t\sin^2(t^2) = $$
$$= 2t$$