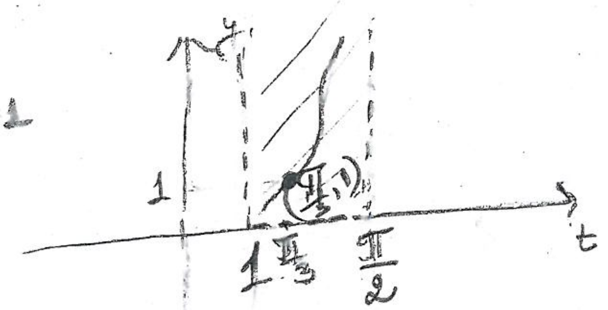
M E T U

Northern Cyprus Campus

Math 219 **Differential Equations**		**I. Short Exam**	**15.03.2015**
Last Name : Name : Student No: KEY	Dept./Sec. : Time : 10: 40 Duration : 50 *minutes*		Signature
3 QUESTIONS			TOTAL 10 POINTS
1	2	3	

Q1 (3 pts.) Find the largest possible rectangular region about $(1, \pi/3)$ in ty-plane, where the conditions of Existence and Uniqueness Theorem are applicable to IVP $\begin{cases} y' = \dfrac{\tan(t)\sqrt{y}}{\ln(t)}, \\ y(\pi/3) = 1. \end{cases}$

$$f(t,y) = \frac{\tan(t)\cdot\sqrt{y}}{\ln(t)}, \quad y(\pi/3)=1$$



1) $\ln(t) \neq 0$

$t \neq 1$

2) $\tan(t) = \dfrac{\sin t}{\cos t}$

$\cos t \neq 0$

$t \neq \frac{\pi}{2}k, \ k \in \mathbb{Z}$

3) $\sqrt{y}$ $\quad y \geq 0$

4) $\dfrac{\partial f}{\partial y} = \dfrac{\tan(t)}{\ln(t)} \cdot \dfrac{1}{2\sqrt{y}}$

$y \neq 0$

$\Downarrow$

$\boxed{y>0, \quad t>1 \text{ and } t<\frac{\pi}{2}}$ $\leftarrow$ IVP has a unique solution on

① ② $1<t<\frac{\pi}{2}$ ③

Q2 (3 pts.) Use the method of Variation of Parameters to solve the given differential equation $y' + \frac{y}{t} = 3\cos(2t)$, $t > 0$.

$$y' + \frac{1}{t}y = 0$$

$$y_h = c e^{-\int \frac{1}{t}dt} = c e^{\ln 1/t} = c \cdot \frac{1}{t}, \quad t > 0 \qquad (1)$$

$$Y(t) = c(t) \cdot \frac{1}{t} \qquad (1)$$

$$\left(c(t) \cdot \frac{1}{t}\right)' + (1/t)\left(c(t) \cdot \frac{1}{t}\right) = 3\cos(2t)$$

$$c'(t) \cdot \frac{1}{t} + \cancel{c(t)\left(-\frac{1}{t^2}\right)} + \cancel{\frac{1}{t^2} c(t)} = 3\cos(2t)$$

$$c'(t) = 3t\cos(2t).$$

$$c(t) = \int 3t\cos(2t)\,dt + C = \frac{3}{2}t\sin(2t) - \frac{3}{2}\int \sin 2t\,dt + C =$$

$$= \frac{3}{2}t\sin(2t) + \frac{1}{2}\frac{3}{2}\cos 2t + C$$

$$(1)$$

$$Y(t) = \frac{3}{2}\sin(2t) + \frac{3}{4t}\cos(2t) + \frac{C}{t}$$

$$y(t) = \underbrace{\frac{3}{2}\sin(2t) + \frac{3}{4t}\cos(2t)}_{\text{a special sol. to non hom,}} + \underbrace{\frac{C}{t}}_{y_h \text{ the sol to hom.}}$$

the general sol.

Q3 (4 pts.) Find an integrating factor and solve the equation $ydt + (ty - y\cos(y))\,dy = 0$.

$y\,dt + (ty - y\cos(y))\,dy = 0$

$\mu(y)y\,dt + \mu(y)(ty - y\cos(y))\,dy = 0.$

$M_y = \mu'(y)y + \mu(y) = N_t = y\mu(y)$

$y\dfrac{d\mu}{dy} = (y-1)\mu$

$\dfrac{d\mu}{\mu} = \dfrac{y-1}{y}dy$

$\int \dfrac{d\mu}{\mu} = \int\left(1-\dfrac{1}{y}\right)dy + c$

$\ln|\mu| = y - \ln|y| + c$

(2) $\mu = c\dfrac{e^y}{y}$ $\quad \xrightarrow{\text{put } c=1} \mu = e^y \cdot 1/y$ $\Rightarrow$ $e^y dt + (te^y - e^y\cos(y))dy = 0$

$M_y = e^y = N_t$

It is exact.

(*)

$\int e^y \cos y\,dy = e^y \sin y - \int e^y \sin y\,dy$

(1) $= e^y \sin y + e^y \cos y - \int e^y \cos y\,dy \Rightarrow$

$\int e^y \cos y\,dy = \frac{e^y}{2}(\sin y + \cos y).$

$\psi_x = M = e^y \quad , \quad \psi_y = N = te^y - e^y\cos y.$

$\psi = \int e^y dt + c(y) = e^y t + c(y)$

$\psi_y = te^y + c'(y) \Rightarrow \cancel{te^y} + c'(y) = \cancel{te^y} - e^y\cos y$

$c'(y) = -e^y\cos y \Rightarrow$

$\Rightarrow c(y) = -\int e^y \cos y\,dy + C \stackrel{(*)}{\Rightarrow} c(y) = -\frac{e^y}{2}(\sin y + \cos y) + C$

$\Rightarrow \psi(t,y) = e^y t - \frac{e^y}{2}(\sin y + \cos y) = c$

(1)