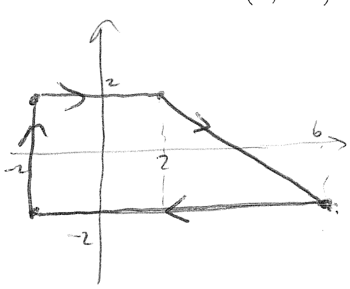
M E T U
Northern Cyprus Campus

	Calculus for Functions of Several Variables Short Exam 3	
Code : *Math 120* Acad.Year: *2012-2013* Semester : *Spring* Date : *21.05.2013* Time : *17:45* Duration : *45 minutes*	Last Name: Name: Student No: Signature: 4+1 QUESTIONS ON 2 PAGES TOTAL 42+4=46 POINTS	

1 (8)	2 (8)	3 (8)	4 (10)	B (4)	KEY		

Show your work! No calculators! Please draw a $\boxed{\text{box}}$ around your answers!
Please do not write on your desk!

1. (8 pts.) Use Green's Theorem to calculate $\int_C \langle 3y+1, \arctan(y^2)\rangle \cdot d\vec{r}$ where C is the perimeter of the trapezoid traversed starting from the point $(-2,-2)$ to $(-2,2)$ to $(2,2)$ to $(6,-2)$ then back to $(-2,-2)$.



$$\int_C \langle 3y+1, \arctan(y^2)\rangle \cdot dr = -\iint_R (0-3)\,dA$$

$$= +3\iint_R dA = 3 \text{ Area}(R)$$

$$= 3\left(4\cdot 4 + \frac{4\cdot 4}{2}\right) = 3\cdot 24 = \boxed{72}$$

2. $(2 \times 4 = 8$ pts.) Evaluate the following limits.

(a) $\lim_{n\to\infty} (e^{120n}+n)^{1/n}$

$$y = (e^{120x}+x)^{1/x}$$

$$\lim_{x\to\infty} \ln y = \lim_{x\to\infty} \frac{\ln(e^{120x}+x)}{x} \overset{(\infty/\infty)}{=} \lim_{x\to\infty} \frac{\frac{1}{e^{120x}+x}\cdot(e^{120x}\cdot 120+1)}{1}$$

$$= \lim_{x\to\infty} \frac{e^{120x}(120+e^{-120x})}{e^{120x}(1+xe^{-120x})} = 120$$

$$\boxed{\lim y = \lim e^{\ln y} = e^{\lim \ln y} = \boxed{e^{120}}}$$

(b) $\lim_{n\to\infty} \left[\ln(2013n+5) - \ln(21n)\right]$

$$= \lim \ln\left(\frac{2013n+5}{21n}\right) = \lim \ln\left(\frac{2013+5/n}{21}\right)$$

$$= \ln \lim \left(\frac{2013+5/n}{21}\right) = \boxed{\ln\left(\frac{2013}{21}\right)}$$

3. ($4 \times 4 = 16$ pts.) Determine if the following series are convergent or divergent. Give brief reasoning.

(a) $\sum_{n=1}^{\infty} n^{1.0001}$

$$\lim_{n\to\infty} n^{1.0001} = +\infty \neq 0$$

TEST FOR DIVERGENCE $\Longrightarrow$ $\boxed{\text{DIVERGEN}}$

(b) $\sum_{n=1}^{\infty} \frac{1}{n^{1.0001} + n + 1.0001}$

limit comparison with $\frac{1}{n^{1.0001}}$

$$\lim \frac{\frac{1}{n^{1.0001}+n+1.0001}}{\frac{1}{n^{1.0001}}} = 1$$

& $\sum_{n=1}^{\infty} \frac{1}{n^{1.0001}}$ CONV. by p test $\Longrightarrow$ $\boxed{\text{CONVERGENT.}}$

(c) $\sum_{n=1}^{\infty} \frac{e^n + \arctan(n!)}{\pi^n}$

$\arctan(n!) < \frac{\pi}{2} \Rightarrow 0 < \frac{e^n + \arctan n!}{\pi^n} < \left(\frac{e}{\pi}\right)^n + \frac{\pi}{2}\left(\frac{1}{\pi}\right)^n$ & $\sum \left(\frac{e}{\pi}\right)^n$ conv & $\sum \frac{\pi}{2}\left(\frac{1}{\pi}\right)^n$ conv

Comparison test $\Longrightarrow$ $\sum_{n=1}^{\infty} \frac{e^n + \arctan(n!)}{\pi^n}$ $\boxed{\text{CONVERGENT}}$

(d) $\sum_{n=1}^{\infty} (-1)^n \frac{n^2}{n^2+1}$

$\lim \frac{n^2}{n^2+1} = 1$ so $\lim (-1)^n \frac{n^2}{n^2+1}$ DNE.

TEST FOR DIVERGENCE $\Longrightarrow$ $\boxed{\text{DIVERGENT}}$

4. (10 pts.) Determine if the series $\sum_{n=2}^{\infty} \frac{1}{n \ln(n)}$ is convergent or divergent. **Give reasoning** by quoting which test you have used and show that all hypothesis hold.

$f(x) = \frac{1}{x \ln x} > 0$, cont. for $x \geq 2$.

$f'(x) = \frac{-(\ln x + 1)}{(x \ln x)^2} < 0$ so f is dec.

$\lim_{x\to\infty} f(x) = 0.$

$\boxed{\text{So we can use Integral Test}}$

$$\int_2^{\infty} \frac{dx}{x \ln x} = \lim_{R\to\infty} \int_2^R \frac{dx}{x \ln x} = \lim_{R\to\infty} \int_{\ln 2}^{\ln R} \frac{du}{u}$$

$$= \lim_{R\to\infty} (\ln \ln R - \ln \ln 2) = +\infty \text{ DNE.}$$

Integral Test $\Longrightarrow$ $\sum_{n=2}^{\infty} \frac{1}{n \ln n}$ is $\boxed{\text{DIVERGENT}}$

5. (Bonus 4 pts.) Find a formula for the general term a_n of the following sequence (starting with a_1).

$$\{-2, \frac{5}{2}, -\frac{10}{6}, \frac{17}{24}, -\frac{26}{120}, \frac{37}{720}, \ldots\}$$

$$\left\{\frac{-2}{1}, \frac{5}{2}, \frac{-10}{1\cdot 2\cdot 3}, \frac{17}{1\cdot 2\cdot 3\cdot 4}, \frac{-26}{1\cdot 2\cdot 3\cdot 4\cdot 5}, \frac{37}{1\cdot 2\cdot 3\cdot 4\cdot 5\cdot 6}, \ldots\right\}$$

$$a_n = \frac{(-1)^n (n^2+1)}{n!} \qquad n = 1, 2, 3, \ldots$$