M E T U
Northern Cyprus Campus

	Calculus for Functions of Several Variables Short Exam 1	
Code : *Math 120* Acad. Year: *2012-2013* Semester : *Fall* Date : *13.03.2013* Time : *17:45* Duration : *40 minutes*	Last Name: Name: KEY Student No: Signature: 5 QUESTIONS ON 2 PAGES TOTAL 42+2=44 POINTS	

1	2	3	4	5			

Show your work! No calculators! Please draw a $\boxed{\text{box}}$ around your answers!
Please do not write on your desk!

1. (a) (2 pts.) Find a unit vector in the direction of $\mathbf{a} = \langle 1, 2, -3 \rangle$.

$$\vec{u} = \frac{1}{\|a\|} \cdot \vec{a} = \frac{1}{\sqrt{1^2+2^2+(-3)^2}} \cdot \langle 1, 2, -3 \rangle = \frac{1}{\sqrt{14}} \cdot \langle 1, 2, -3 \rangle$$

(b) (4 pts.) Give a vector equation for the line through the point $(1, 2, 3)$ that is parallel to the line $\langle -1 - 4t, 2t - 1, 5 + t \rangle$. $\rightarrow v = \langle -4, 2, 1 \rangle$

$$r(t) = (1,2,3) + t(-4,2,1) = \langle -4t+1, 2t+2, t+3 \rangle \quad t \in \mathbb{R}$$

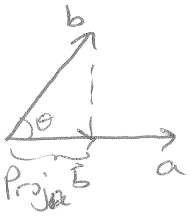
(c) (4 pts.) Find an equation of the plane through the point $(2, 3, 0)$ and perpendicular to the vector $\langle 2, -3, 4 \rangle$.

$$2x - 3y + 4z = 2 \cdot 2 + (-3)(3) + 4 \cdot 0$$

$$2x - 3y + 4z = -5$$

2. ($3 \times 4 = 12$ pts.) Let $\mathbf{a} = \langle 1, 2, -3 \rangle$ and $\mathbf{b} = \langle 5, -3, 0 \rangle$.

(a) Find **the scalar projection** of the vector $\mathbf{b}$ onto the vector $\mathbf{a}$.



$$\mathrm{Comp}_{\vec{a}} \vec{b} = \|b\| \cdot \cos\theta = \frac{a \cdot b}{\|a\|} = \frac{1 \cdot 5 + 2 \cdot (-3) + (-3) \cdot 0}{\sqrt{1^2+2^2+(-3)^2}} = \frac{5-6}{\sqrt{14}} = \frac{-1}{\sqrt{14}}$$

(b) Find **the vector projection** of the vector $\mathbf{b}$ onto the vector $\mathbf{a}$.

$$\mathrm{Proj}_{\vec{a}} \vec{b} = \mathrm{Comp}_{\vec{a}} \vec{b} \cdot \frac{\vec{a}}{\|a\|} = \frac{a \cdot b}{a \cdot a} \cdot a = \frac{-1}{14} \cdot \langle 1, 2, -3 \rangle = \left\langle \frac{-1}{14}, \frac{-1}{7}, \right.$$

(c) Find **the orthogonal projection** of the vector $\mathbf{b}$ onto the vector $\mathbf{a}$.

$$\mathrm{Orth}_{\vec{a}} \vec{b} = b - \mathrm{Proj}_{\vec{a}} b = \langle 5, -3, 0 \rangle - \left\langle \frac{-1}{14}, \frac{-1}{7}, \frac{3}{14} \right\rangle$$

$$= \left\langle 5 + \frac{1}{14}, -3 + \frac{1}{7}, 0 - \frac{3}{14} \right\rangle$$

$$= \left\langle \frac{71}{14}, \frac{-20}{7}, \frac{-3}{14} \right\rangle$$

3. (10 pts.) Determine whether the given lines are parallel, intersecting, or skew. If they intersect, find the intersection point. Show your work.

$$L_1 : x = t + 1,\ y = 2t + 1,\ z = 3t + 2 \longrightarrow v_1 = \langle 1, 2, 3\rangle$$
$$L_2 : x = s + 5,\ y = -s + 3,\ z = s + 10 \longrightarrow v_2 = \langle 1, -1, 1\rangle$$

$L_1 \parallel L_2 \Leftrightarrow v_1 \parallel v_2$. $\underline{v_1 \parallel v_2\ ?}$ $v_1 = k \cdot v_2 \Leftrightarrow \begin{array}{l} 1 = k\cdot 1 \\ 2 = k\cdot(-1) \\ 3 = k\cdot 1 \end{array}$ No soln.

$\Rightarrow v_1$ is not parallel to $v_2 \Rightarrow L_1$ is not parallel to L_2.

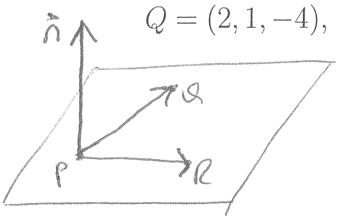
$\underline{L_1 \cap L_2\ ?}$
$$\begin{aligned} t+1 &= s+5 \\ 2t+1 &= -s+3 \\ 3t+2 &= s+10 \end{aligned} \quad \xrightarrow{(1\&2)} \quad 3t+2 = 8 \Rightarrow 3t = 6 \Rightarrow t = 2$$
$\Rightarrow s = -2$

Check 3rd eqn. $3\cdot 2 + 2 \overset{?}{=} -2 + 10$
YES

$\Rightarrow (s,t) = (2,-2)$ gives an intersection

$L_1 \cap L_2 = \{(3,5,8)\}$

4. (10 pts.) Find an equation of the plane that passes through the points $P = (1, 2, 3)$, $Q = (2, 1, -4)$, and $R = (0, 3, 0)$



$$\vec{n} = \vec{PQ} \times \vec{PR} = \langle 1,-1,-7\rangle \times \langle -1, 1, -3\rangle$$
$$= \langle -1, 1, 7\rangle \times \langle 1, -1, 3\rangle$$
$$= \begin{vmatrix} i & j & k \\ -1 & 1 & 7 \\ 1 & -1 & 3 \end{vmatrix} = \langle 3+7, -(-3-7), 1-1 \rangle$$
$$= \langle 10, 10, 0\rangle$$

instead might as well use $\vec{n} = \langle 1, 1, 0\rangle$

Then $\pi : x + y + 0\cdot z = d = 1\cdot 1 + 1\cdot 2 + 0\cdot 3$

$$\boxed{x + y = 3}$$

5. (2 pts.) **Bonus Question** Write the schedule for any of the recitation sections for Math 120 this semester.

- Monday $13^{40} - 15^{30}$
- Tuesday $15^{40} - 17^{30}$
- Wednesday $8^{40} - 10^{30}$
- Thursday $15^{40} - 17^{30}$