M E T U
Northern Cyprus Campus

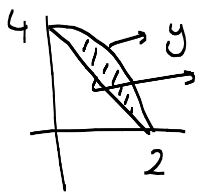
	Calculus with Analytic Geometry Short Exam 3	
Code : *Math 119* Acad.Year: *2013-2014* Semester : *Summer* Date : *12.08.2014* Time : *18:00* Duration : *30 minutes*	Last Name: Name: KEY Student No: Signature: 3 QUESTIONS 2 PAGES TOTAL 20 POINTS	

1(8)	2(6)	3(6)			

Show your work! No calculators! Please draw a $\boxed{\text{box}}$ around your answers!
Please do not write on your desk!

1. (2+3+3 = 8 pts.) Let R be the region bounded by the curves $y = 4 - x^2$ and $y = 4 - 2x$. Let S be the solid obtained by rotating the region R about the line $y = -1$.

   (a) Find *the area of R.*



$$\text{Area of } R = \int_0^2 (4 - x^2 - (4 - 2x))\,dx$$

$$= \int_0^2 (2x - x^2)\,dx = x^2 - \frac{x^3}{3} \Big|_0^2$$

$$= 4 - \frac{8}{3} - 0 = \frac{4}{3}$$

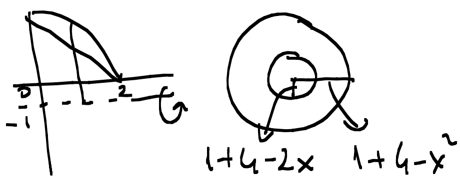
$4 - x^2 = 4 - 2x$

$\Leftrightarrow x^2 - 2x = 0$

$\Leftrightarrow x(x-2) = 0$

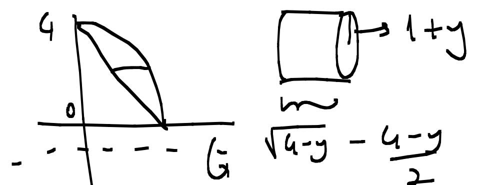
$\Leftrightarrow x = 0$ or $x = 2$

   (b) **Write the integral** to obtain *the volume of S* **using the method of disks or washers**. DO NOT EVALUATE THIS INTEGRAL.



$$\int_0^2 \pi \left[(4 - x^2 + 1)^2 - (4 - 2x + 1)^2 \right] dx$$

   (c) **Write the integral** to obtain *the volume of S* **using the method of shells**. DO NOT EVALUATE THIS INTEGRAL.



$$\int_0^4 2\pi (1+y) \left(\sqrt{4-y} - \frac{4-y}{2} \right) dy$$

$4 - x^2 = y \Rightarrow x = \sqrt{4-y}$

$4 - 2x = y \Rightarrow x = \frac{4-y}{2}$

2. ($3 \times 2 = 6$ pts.) Evaluate the limit, if it exists.

(a) $\lim_{x\to 0^+} \sin x \ln x = \lim_{x\to 0^+} \frac{\ln x}{\frac{1}{\sin x}} \overset{L'H}{=} \lim_{x\to 0^+} \frac{\frac{1}{x}}{-\frac{\cos x}{\sin^2 x}}$

$= \lim_{x\to 0^+} -\frac{\sin^2 x}{x\cos x} = \lim_{x\to 0^+} -\frac{\sin x}{x} \cdot \frac{\sin x}{\cos x} = -1 \cdot \frac{0}{1} = 0$

(b) $\lim_{x\to 0} \frac{1}{\sin x} \int_0^{\sin(2x)} \cos(5t)\, dt = \lim_{x\to 0} \frac{\int_0^{\sin 2x} \cos(5t)\, dt}{\sin x}$

$\overset{\substack{L'H \\ FTC}}{=} \lim_{x\to 0} \frac{\cos(5\sin 2x)\cdot \cos 2x \cdot 2}{\cos x} = \frac{1 \cdot 1 \cdot 2}{1} = 2$

(c) $\lim_{x\to\infty} x^{\frac{1}{x}} = \lim_{x\to\infty} e^{\ln x^{(1/x)}} = e^{\lim_{x\to\infty} \frac{\ln x}{x}} \overset{L'H}{=} e^{\lim_{x\to\infty} \frac{\frac{1}{x}}{1}}$

$= e^0 = 1.$

3. ($2 \times 4 = 8$ pts.) Find the following indefinite integrals.

(a) $\int \sin(x)\, 2^{\cos(x)}\, dx = \int -2^u\, du = -\frac{2^u}{\ln 2} + c$

$\cos x = u$
$-\sin x\, dx = du$

$= -\frac{2^{\cos x}}{\ln 2} + c$

(b) $\int \arctan(t)\, dt = t\arctan t - \int \frac{t}{1+t^2}\, dt = t\arctan t - \int \frac{ds}{2s}$

$\arctan t = u$
$\frac{1}{1+t^2} = du$
$dt = dv$
$t = v$

$s = 1+t^2$
$ds = 2t\, dt$

$= t\arctan t - \frac{1}{2}\ln|s| + c$

$= t\arctan t - \frac{1}{2}\ln(1+t^2) + c$

DID YOU WRITE YOUR NAME AND ID NUMBER ON THE PAPER?