


# M E T U

## Northern Cyprus Campus

Calculus with Analytic Geometry Short Exam 1A			
Code : <i>Math 119</i>	Last Name:		
Acad. Year: <i>2013-2014</i>	Name:		Student No:
Semester : <i>Spring</i>			
Date : <i>19.03.2013</i>			
Time : <i>17:45</i>			
Duration : <i>35 minutes</i>			
4 QUESTIONS 2 PAGES TOTAL 21 POINTS			
1(4)	2(4)	3(9)	4(4)

**Show your work! No calculators! Please draw a box around your answers!**  
**Please do not write on your desk!**

1. ( $2 \times 2 = 4$  pts.) Evaluate the limit, if it exists. **Give reasoning.**

$$(a) \lim_{x \rightarrow 1} \frac{x^2 + 2x}{x^2 - 4} = \frac{\lim_{x \rightarrow 1} x^2 + 2x}{\lim_{x \rightarrow 1} x^2 - 4} = \frac{3}{-3} = -1$$

$\downarrow$   
 (continuity)

$$\left( \lim_{x \rightarrow 1} x^2 - 4 = -3 \neq 0 \right)$$

$$(b) \lim_{x \rightarrow 1} \frac{|x^3 + x^2 - 5x + 3|}{x - 1} = \lim_{x \rightarrow 1} \frac{|(x-1)^2| |x+3|}{x-1} = \lim_{x \rightarrow 1} \frac{(x-1)^2 |x+3|}{x-1}$$

$$= \lim_{x \rightarrow 1} (x-1) |x+3| = 0$$

$\downarrow$   
 (continuity)

2. (4 pts.) Write an equation of the tangent line to the curve  $y = \frac{2\sin(x) + 3}{x^2 + 1}$  at  $x = 0$ .

$$\text{at } x = 0, y = 3$$

$$y' = \frac{2 \cos x (x^2 + 1) - 2x (2 \sin x + 3)}{(x^2 + 1)^2}$$

$$x = 0 \Rightarrow y' = 2$$

eqn of tangent line is

$$y - 3 = 2(x - 0) \Leftrightarrow \boxed{y = 2x + 3}$$

3. ( $9 \times 1 = 9$  pts.) Determine whether the given statement is true or false. Indicate your answers by typing **TRUE** or **FALSE**. No explanations required.

(a) If  $f$  is continuous at  $a$  then it is differentiable at  $a$ .

FALSE

(b) If  $\lim_{x \rightarrow a} f(x) = 0$  and  $\lim_{x \rightarrow a} g(x) = 0$ , then  $\lim_{x \rightarrow a} \frac{f(x)}{g(x)}$  does not exist.

FALSE

(c) If  $\lim_{x \rightarrow a} f(x)g(x)$  exists then  $\lim_{x \rightarrow a} f(x)g(x) = f(a)g(a)$ .

FALSE

(d) If  $f$  is differentiable at  $a$  then it has a limit at  $a$ .

TRUE

(e)  $|x^2 + 2|$  is differentiable everywhere.

TRUE

(f)  $\lim_{x \rightarrow 1} \frac{x^2 + 2x - 2}{x^2 + 8x - 7} = \frac{\lim_{x \rightarrow 1} x^2 + 2x - 2}{\lim_{x \rightarrow 1} x^2 + 8x - 7}$

TRUE

(g)  $\lim_{x \rightarrow 0} \frac{1}{x} = +\infty$

FALSE

(h)  $\frac{x^2 - 1}{x - 1} = x + 1$

FALSE

(i)  $\lim_{x \rightarrow 2} \frac{x^2 - 1}{x - 1} = \lim_{x \rightarrow 2} x + 1$

TRUE

4. ( $2 + 2 = 4$  pts.) Consider the following items.

(1)  $\epsilon > 0$  (2)  $\delta > 0$  (3)  $0 < |x - c| < \delta$  (4)  $|f(x) - L| < \epsilon$  (5)  $|f(x) - L| > \epsilon$  (6) but (7) such that (8) for all (9) there is some (10) there is some  $x$  (11) whenever

(a) Write the numerical sequence for the definition of  $\lim_{x \rightarrow c} f(x) = L$ .

8 - 1 - 9 - 2 - 7 - 4 - 11 - 3

(b) Write the numerical sequence to form a rigorous assertion that  $\lim_{x \rightarrow c} f(x) \neq L$ .

9 - 1 - 7 - 8 - 2 - 10 - 7 - 3 - 6 - 5