

M E T U

Northern Cyprus Campus

Calculus with Analytic Geometry Short Exam 2		
Code : <i>Math 119</i>	Last Name:	<div style="font-size: 2em; opacity: 0.5; transform: rotate(-45deg); display: inline-block;">METU</div>
Acad. Year: <i>2012-2013</i>	Name:	
Semester : <i>Summer</i>	Signature:	Student No:
Date : <i>15.07.2013</i>	4 QUESTIONS ON 2 PAGES	
Time : <i>17:45</i>	TOTAL 10 POINTS	
Duration : <i>30 minutes</i>		
1	2	3
4		

Show your work! No calculators! Please draw a box around your answers!

Please do not write on your desk!

1. (1 pt.) Determine whether the given statement is true or false. If it is true, explain why. If it is false, explain why or give an example that disproves the statement.

Every critical point is either a local maximum or a local minimum.

FALSE.

eg: $f(x) = x^3$.

0 is a critical number of f but f is strictly increasing so it does not have any local max. or min.

2. (3 pts.) Use linear approximation, i.e. the tangent line, to approximate $\sqrt[3]{-0.9}$

Let $f(x) = \sqrt[3]{x}$

To approximate $\sqrt[3]{-0.9}$, use linearization $L(x)$ of $f(x)$

at $a = -1$,

$$L(x) = f(-1) + f'(-1)(x - (-1)) = f(-1) + f'(-1)(x + 1)$$

$$f(-1) = \sqrt[3]{-1} = -1$$

$$f'(x) = \frac{1}{3\sqrt[3]{x^2}} \Rightarrow f'(-1) = \frac{1}{3}$$

$$\text{So } \sqrt[3]{-0.9} = f(-0.9) \approx L(-0.9) = -1 + \frac{1}{3}(-0.9 + 1)$$

$$= \boxed{-\frac{29}{30}}$$

3. (3 pts.) Let $f(x) = \frac{x^3}{x^2 - 3}$.

Find the intervals of concavity of $f(x)$ and the inflection points of the graph of $f(x)$.

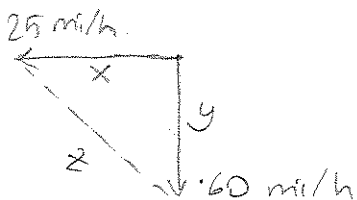
$$f''(x) = \frac{6x(x^2 + 9)}{(x^2 - 3)^3} \quad f''(x) = 0 \Leftrightarrow x = 0$$

	$-\sqrt{3}$	0	$\sqrt{3}$
f''	$-$	$+$	$-$
f	\cap	\cup	\cap

f is concave up on $(-\sqrt{3}, 0) \cup (\sqrt{3}, \infty)$
 down on $(-\infty, 0) \cup (0, \sqrt{3})$

Only inflection point is $(0, 0)$. ($f''(0) = 0$ and concavity changes at 0)

4. (3 pts.) Two cars start moving from the same point. One travels south at 60 mi/h and the other travels west at 25 mi/h . At what rate is the distance between the cars increasing two hours later?



$$z^2 = x^2 + y^2$$

$$\frac{dx}{dt} = 25 \quad \frac{dy}{dt} = 60$$

$$t = 2 \Rightarrow x = 50, y = 120, z = 130$$

$$\frac{dz}{dt} \Big|_{t=2} = ?$$

$$z^2 = x^2 + y^2 \Rightarrow 2z \frac{dz}{dt} = 2x \frac{dx}{dt} + 2y \frac{dy}{dt}$$

$$t = 2 \Rightarrow 130 \frac{dz}{dt} \Big|_{t=2} = 50 \cdot 25 + 60 \cdot 120$$

$$\Rightarrow \frac{dz}{dt} \Big|_{t=2} = \frac{125 + 720}{13} = \boxed{65 \text{ mi/h}}$$