M E T U
Northern Cyprus Campus

	Calculus with Analytic Geometry Short Exam 3	
Code : *Math 119* Acad.Year : *2012-2013* Semester : *Spring* Date : *21.05.2013* Time : *17:45* Duration : *45 minutes*	Last Name: Name: Student No: Signature:	4+1 QUESTIONS ON 2 PAGES TOTAL 42+4=46 POINTS

1 (8)	2 (10)	3 (16)	4 (8)	B (4)			

Show your work! No calculators! Please draw a box around your answers!

Please do not write on your desk!

1. (8 pts.) Use integration by parts to evaluate $\int_1^9 \sqrt{t}\ln t\,dt$.

$u = \ln t \Rightarrow \frac{1}{t}dt = du$

$\sqrt{t}\,dt = dv \Rightarrow \frac{2}{3}t^{3/2} = v$

$$\frac{2}{3}t^{3/2}\ln t\Big|_1^9 - \int_1^9 \frac{2}{3}t^{3/2}\cdot\frac{1}{t}dt = \frac{2}{3}3^3\ln 9 - 0 - \left(\frac{4}{9}t^{3/2}\Big|_1^9\right)$$

$$= 4\cdot 9\ln 3 - \left(\frac{4}{9}3^3 - \frac{4}{9}\right) = 36\ln 3 - 12 + \frac{4}{9}$$

2. (10 pts.) What is the correct form of the partial fraction decomposition of the following?

DO NOT CALCULATE THE COEFFICIENTS.

$$\frac{x^4+1}{(x^3+x)^2(x^2-2x-3)}$$

$$(x^3+x)^2(x^2-2x-3) = x^2(x^2+1)^2(x-3)(x+1)$$

$$\frac{x^4+1}{(x^3+x)^2(x^2-2x-3)} = \frac{A}{x-3} + \frac{B}{x+1} + \frac{C}{x} + \frac{D}{x^2} + \frac{Ex+F}{x^2+1} + \frac{Gx+H}{(x^2+1)^2}$$

3. $(2 \times 8 = 16$ pts.) Evaluate the following indefinite integrals.

(a) $\int \frac{x+1}{x^2+1}\,dx$.

$= \int \frac{x}{x^2+1}\,dx + \int \frac{1}{x^2+1}\,dx$

$(x^2+1 = u)$

$= \int \frac{du}{2u} + \arctan x = \frac{1}{2}\ln(x^2+1) + \arctan x + C$

$\frac{\ln|u|}{2} + C$

(b) $\int \tan^3(x)\sec^5(x)\,dx$. $= \int \tan^2 x \sec^4 x \sec x \tan x\,dx$

$u = \sec x$

$du = \sec x \tan x\,dx$

$\tan^2 x = \sec^2 x - 1$

$= \int (u^2-1)u^4\,du = \int u^6 - u^4\,du$

$= \frac{u^7}{7} - \frac{u^5}{5} + C$

$= \frac{\sec^7(x)}{7} - \frac{\sec^5(x)}{5} + C$

4. (8 pts.) Use logarithmic differentiation to find $\frac{dy}{dx}$ where $y = x^{e^x}$.

$\ln y = \ln(x^{e^x}) = e^x \ln x$

$\Rightarrow \frac{y'}{y} = e^x \ln x + \frac{e^x}{x} \Rightarrow y' = x^{e^x}\left(e^x \ln x + \frac{e^x}{x}\right)$

5. Bonus Question (4 pts. No partial credits.) Evaluate $\int_0^1 \underbrace{\ln(x^2+15)}_{u}\underbrace{dx}_{dv}$. $= I$

$du = \frac{2x\,dx}{x^2+15}$ $\quad v = x$

$I = x\ln(x^2+15)\Big|_0^1 - 2\int_0^1 \frac{x^2+15-15}{x^2+15}\,dx = \ln 16 - 2\int_0^1 \left(1 - \frac{15}{x^2+15}\right)dx$

$= \ln 16 - 2\left(x - \frac{15}{\sqrt{15}}\left(\arctan\frac{x}{\sqrt{15}}\right)\right)\Big|_0^1$

$= \ln 16 - 2\left(1 - \sqrt{15}\arctan\frac{1}{\sqrt{15}}\right)$.