# M E T U
# Northern Cyprus Campus

| | Calculus with Analytic Geometry<br>Short Exam 2 | |
|---|---|---|
| Code : *Math 119*<br>Acad.Year: *2012-2013*<br>Semester : *Spring*<br>Date : *15.04.2013*<br>Time : *17:45*<br>Duration : *35 minutes* | Last Name:<br>Name: KEY Student No:<br>Signature:<br><br>4+1 QUESTIONS ON 2 PAGES<br>TOTAL 42+4=46 POINTS | |

| 1 | 2 | 3 | 4 | 5 | | | |
|---|---|---|---|---|---|---|---|
| | | | | | | | |

**Show your work! No calculators! Please draw a box around your answers!**
**Please do not write on your desk!**

1. (8 pts.) Use linear approximation, i.e. the tangent line, to approximate $\sqrt[3]{28}$.

Let $f(x) = \sqrt[3]{x}$, $L(x) = f(27) + f'(27)(x-27)$

Then $\sqrt[3]{28} \approx L(28)$.

$f(27) = \sqrt[3]{27} = 3$

$f'(x) = \frac{1}{3} x^{-2/3}$

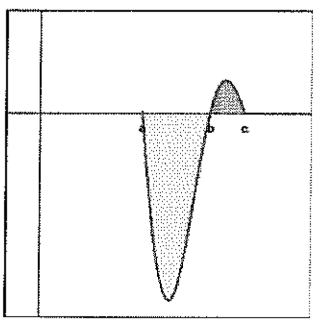
$f'(27) = \frac{1}{3} \cdot \frac{1}{9} = \frac{1}{27}$

$L(x) = 3 + \frac{1}{27}(x-27)$

$\sqrt[3]{28} \approx L(28) = 3 + \frac{1}{27}(28-27)$

$= \boxed{3 + \frac{1}{27}}$

2. ($4 \times 2 = 8$ pts.) Suppose the region on the left in the figure has area 38, and the region on the right has area 4. Using the graph of f(x) in the figure, find the following integrals.



- $\int_a^b f(x)\,dx = \boxed{-38}$
- $\int_b^c f(x)\,dx = \boxed{4}$
- $\int_a^c f(x)\,dx = \int_a^b f(x)dx + \int_b^c f(x)dx = -38 + 4 = \boxed{-34}$
- $\int_a^c |f(x)|\,dx = \int_a^b -f(x)\,dx + \int_b^c f(x)\,dx = +38 + 4 = \boxed{42}$

3. (8 pts.) Find the minimum distance of the parabola $x = y^2$ to the point $(\frac{1}{2}, 2)$.

If $(x, y)$ is on the parabola then $x = y^2$,

i.e $(x, y) = (y^2, y)$.

distance between $(\frac{1}{2}, 2)$ and a point $(y^2, y)$ of the parabola is the function $d$:

$d(y) = \sqrt{(y^2 - \frac{1}{2})^2 + (y-2)^2} = \sqrt{y^4 - 4y + \frac{17}{4}}$

Absolute extrema of $d(y)$ and $D(y) = y^4 - 4y + \frac{17}{4}$ are the same. So consider $D(y)$.

$D'(y) = 4y^3 - 4$ $\quad D'(y) = 0 \iff y^3 = 1 \iff y = 1$.

| | 1 | |
|---|---|---|
| $D'$ | $-$ | $+$ |

Since 1 is the only critical pnt of $D$, and $D$ is cont., $y = 1$ is the absolute min. of $D$.

So the min. distance occurs at $y = 1$ with value

$d(1) = \sqrt{1 - 4 + \frac{17}{4}} = \frac{\sqrt{5}}{2}$

4. $(8 + 8 + 2 = 18$ pts.) Consider the following table.

| $x$ | 3 | 6 | 9 | 12 | 15 |
|---|---|---|---|---|---|
| $f(x)$ | 2 | 1 | 5 | 4 | 9 |

3 6 9 12

- Use this data and a ~~left~~ right-endpoint Riemann sum to estimate the integral
  $\int_{\cancel{0}\,3}^{15} f(x)\,dx \approx 3(f(6) + f(9) + f(12) + f(15)) = 3.19 = 57$
- Use this data and a left-endpoint Riemann sum to estimate the integral
  $\int_{\cancel{0}\,3}^{15} f(x)\,dx \approx 3(f(3) + f(6) + f(9) + f(12)) = 3.12 = 36$
- Use the above to estimate the integral by taking the average.
  $\int_{\cancel{0}\,3}^{15} f(x)\,dx \approx \frac{57 + 36}{2} = \frac{93}{2}$

5. (Bonus)(4 pts.) The function $f(x) = x^{2/3}$ passes from the points $(-1, 1)$ and $(1, 1)$, and has the derivative $f'(x) = \frac{2}{3\sqrt[3]{x}}$. But, clearly, $f'(x) = 0$ does not have a solution. Is this a contradiction to the Mean Value Theorem?

**Briefly explain.**

MVT says if $f$ is cont on $[a,b]$, diff'ble on $(a,b)$ then $\exists\, c \in (a,b)$ st $f'(c) = \frac{f(b) - f(a)}{b - a}$.

Here $0 \in (-1, 1)$ but $f$ is not diff'ble at 0.

So MVT does not apply on an interval containing 0 for this function.