M E T U
Northern Cyprus Campus

<table>
<tr><td colspan="2">Calculus With Analytic Geometry
Short Exam 1</td></tr>
<tr><td>Code : Math 119
Acad.Year : 2011-2012
Semester : Summer
Date : 27.7.2012
Time : 16:40
Duration : 30 minutes</td><td>Last Name: Name:
Department: Student No:
Section: Signature:
Recitation:
4 QUESTIONS ON 2 PAGES
TOTAL 50 POINTS</td></tr>
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1	2	3	4			

Show your work! No calculators! Please draw a $\boxed{\text{box}}$ around your answers!
Please do not write on your desk!

1. (12 pts.) Evaluate the following limits if they exist. Give reasoning.

(a) $\lim\limits_{x\to\infty} \sqrt{x}\sin\left(\frac{1}{x}\right) = \lim\limits_{x\to\infty} \sqrt{x}\sin\left(\frac{1}{x}\right)\cdot\frac{(1/x)}{(1/x)} = \lim\limits_{x\to\infty}\frac{1}{\sqrt{x}}\cdot\frac{\sin(1/x)}{(1/x)}$

$= \left(\lim\limits_{x\to\infty}\frac{1}{\sqrt{x}}\right)\left(\lim\limits_{x\to\infty}\frac{\sin(1/x)}{(1/x)}\right) = 0.1 = \boxed{0}$

(b) $\lim\limits_{x\to-\infty} \frac{\sqrt{9x^6 - x}}{x^3+1} = \lim\limits_{x\to-\infty}\frac{|x^3|\sqrt{9-1/x^5}}{x^3(1+1/x^3)} = \lim\limits_{x\to\infty}\frac{-\cancel{x^3}\sqrt{9-\cancel{1/x^5}}^{\,0}}{\cancel{x^3}(1+\cancel{1/x^3}^{\,0})}$

$|x^3| = -x^3$

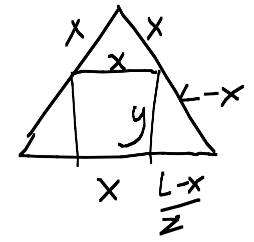
$(x\to-\infty \Rightarrow x<0)$

$= -\frac{\sqrt{9}}{1} = \boxed{-3}$

(c) $\lim\limits_{x\to\infty}(\sqrt{4x^2+x}-2x)\cdot\frac{\sqrt{4x^2+x}+2x}{\sqrt{4x^2+x}+2x} = \lim\limits_{x\to\infty}\frac{\cancel{4x^2}+x-\cancel{4x^2}}{|x|(\sqrt{4+1/x}+2)}$

$= \lim\limits_{x\to\infty}\frac{\cancel{x}}{\cancel{x}(\sqrt{4+\cancel{1/x}^{\,0}}+2)} = \frac{1}{\sqrt{4}+2} = \boxed{\frac{1}{4}}$

2. (10 pts.) Find the dimensions of the rectangle of largest area that can be inscribed inside an equilateral triangle of side L if one side of the rectangle lies on the base of the triangle.



$\tan\frac{\pi}{3} = \sqrt{3} = \frac{y}{\frac{L-x}{2}} \Rightarrow y = \frac{\sqrt{3}}{2}(L-x)$

Area of the rectangle $= xy = \frac{\sqrt{3}}{2}x(L-x) = A(x)$

$A'(x) = \frac{\sqrt{3}}{2}(L-2x)$ $\quad A'(x) = 0$ iff $\boxed{x = \frac{L}{2}} \Rightarrow \boxed{y = \frac{\sqrt{3}}{4}L}$

$A''(x) = -\sqrt{3} \Rightarrow A''\left(\frac{L}{2}\right) = -\sqrt{3} < 0 \Rightarrow A\left(\frac{L}{2}\right)$ is the only local max.

$\Rightarrow$ abs. max.

3. (8 pts.) Find the linear approximation of $f(x) = \sqrt[3]{x}$ at $a = 8$, and use it to approximate the number $\sqrt[3]{7}$.

$f(8) = 2$, $f'(x) = \frac{1}{3\sqrt[3]{x^2}} \Rightarrow f'(8) = \frac{1}{12}$

$f(x) \approx L(x) = f(8) + f'(8)(x-8) = 2 + \frac{1}{12}(x-8)$

$\Rightarrow \sqrt[3]{7} = f(7) \approx L(7) = 2 + \frac{1}{12}(7-8) = 2 - \frac{1}{12} = \frac{23}{12}$

4. (20 pts.) Find **the absolute maximum** and **the absolute minimum** values of $f(x) = \frac{3x^2}{2} - 7x - \frac{4}{x}$ on each of the indicated intervals. Write D.N.E. as your answer if any of these values doesn't exist.

$f'(x) = 3x - 7 + \frac{4}{x^2} = \frac{3x^3 - 7x^2 + 4}{x^2} = \frac{(x-1)(x-2)(3x+2)}{x^2}$

$f'(x) = 0$ iff $x = -\frac{2}{3}, 1, 2$

$f'(x)$ does not exist iff $x = 0$

	$-2/3$		0		1		2	
f'	$-$		$+$	$\cancel{8}$	$+$		$-$	$+$
f	↘		↗		↗		↘	↗

(a) $[0, 3/2]$

critical pnts: $0, 1$, end points $0, \frac{3}{2}$

$\lim_{x\to 0^+} f(x) = -\infty \Rightarrow$ $\boxed{\text{abs min DNE}}$

$f(1) = -\frac{19}{2} > f(\frac{3}{2}) = -\frac{335}{24} \Rightarrow$ $\boxed{\text{abs. max} = -\frac{19}{2}}$

(b) $[1/2, 3]$

critical pnts: $1, 2$ end pnts: $\frac{1}{2}, 3$ $\boxed{\text{abs max} = -\frac{19}{2}}$

$f(\frac{1}{2}) = -\frac{89}{8} \approx -11.3$ $f(2) = -10$

$f(1) = -\frac{19}{2} = -9.5$ $f(3) = \frac{-53}{6} \approx -8.83$ $\boxed{\text{abs min} = -\frac{89}{8}}$

(c) $(-3, -1)$

f is strictly decreasing on the interval $(-3, -1)$, so abs. extrema occur on the boundary. But $(-3, -1)$ is an open interval (i.e does not have end points!) So $\boxed{\text{abs. max DNE}}$ $\boxed{\text{abs min DNE}}$

(d) $[-1, 1]$

$\lim_{x\to 0^+} f(x) = -\infty \Rightarrow$ $\boxed{\text{abs min DNE}}$, $\lim_{x\to 0^-} f(x) = \infty \Rightarrow$ $\boxed{\text{abs max DNE}}$