RESEARCH PROPOSAL Anar Dosi(ev)
Quantum Cones and their Applications, Noncommutative geometry

My research interests are mainly in noncommutative analysis. This is a modern branch of mathematical, more precisely functional analysis, whose classical foundation goes back to the known calculus that we have been teaching at the universities. The central idea of noncommutative analysis is to develop a calculus over elements of a certain noncommutative base algebra. Classical multivariable calculus can be quantized on by replacing commuting variables $x_1, \ldots, x_n$ by noncommuting (for instance 2×2) matrices $A_1, \ldots, A_n$. These are called "quantum" or "noncommutative variables". This type of model certainly involves lots of abstract algebra, topology, geometry and functional analysis techniques. In order to have a strong and unique mathematical language of quantization there was recently created so called quantum functional analysis. The latter in turn can be treated as the basic language of quantum physics. To have a comprehensive mathematical model of quantum physics, it is also necessary to consider the linear spaces of unbounded Hilbert space operators or "noncommutative variable spaces". It is well known that if we interpret the time dependent variables as infinite matrices rather than functions, then quantum phenomena can be deduced from the equations of Newtonian physics. This is the basic idea of Heisenberg's "matrix mechanics". These "quantum variables" have provided functional analysis with new constructions, methods and problems. Being a new and modern branch of functional analysis, quantum functional analysis is covered by normed and polynormed topics, as in the classical theory.

In the last three years I got various results on quantum polynormed spaces, namely, representation theorems, duality theory, quantum systems, local operator algebras, quantum bornology. These were successfully applied to quantum moment problems. My future plan is to investigate quantum cones in their general setting and provide applications to quantum information theory. Quantum cones probably will be the main fundamental concept of the whole quantum functional analysis and they will play the crucial role in the applications. Our approach will lead to a right understanding of minimal and maximal quantum (in particular, operator) systems, which in turn creates a solid foundation to characterize the "partially entanglement breaking maps" encountered in quantum information theory.

The last direction of interest deals with a new approach to noncommutative algebraic geometry. One of the principal foundations of noncommutative algebraic geometry is to extend the concept of affine schemes to noncommutative rings. As in the commutative case the main motivation for this is to represent a non- commutative ring as the ring of "functions" over its spectrum. The classical result, which is due to I. M. Gelfand, asserts that a commutative Banach algebra can be realized as the algebra of continuous functions over the space of all its maximal ideals modulo its Jacobson radical. For the commutative rings this result has been generalized in the following way, which is due to A. Grothendieck. A commutative ring A is the ring of all global sections $\Gamma(\operatorname{Spec}(A), \mathcal{O})$ of the structure sheaf $\mathcal{O}$ of the ring A over its scheme $\operatorname{Spec}(A)$ (the space of all prime ideals of A) up to an isomorphism. The construction of the relevant schemes, and "functions" over them, for the noncommutative ring requires extraordinary efforts. The problem can not be solved in a unique framework based on a certain special category of objects and morphisms. All this diversity reflects in various methods and constructions picked up in noncommutative geometry. Lie algebra methods allow us to handle the problem. As shown in many investigations the most reliable case for the relevant scheme theory to be built up is the class of Lie-nilpotent rings. This proposal has been partially supported in Kapranov's theory (1998) of NC-schemes. Another motivation for this direction is to use an operator approach to noncommutative schemes, without any sheaf construction as classically done. The operator realization of many noncommutative algebras is the well known fact. In this direction I intend to develop a noncommutative regularity in the general purely algebraic case, and propose a new approach to noncommutative spectra which is based on concrete operator realization of an abstract regularity in a Lie-complete ring. The approach proposed presents an interest in the commutative case as well.