M E T U
Northern Cyprus Campus

	Calculus With Analytic Geometry Short Exam 3	
Code : *Math 119* Acad.Year: *2011-2012* Semester : *Spring* Date : *22.5.2012* Time : *18:45* Duration : *45 minutes*	Last Name: Name: Department: Student No: Section: Signature:	5 QUESTIONS ON 2 PAGES TOTAL 50 POINTS

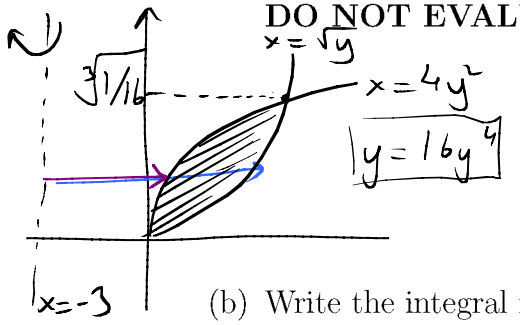
1	2	3	4	5			

Show your work! No calculators! Please draw a box around your answers!

Please do not write on your desk!

1. (2×6 = 12 pts.)) Let R be the region in the first quadrant bounded by the curves $x = \sqrt{y}$ and $x = 4y^2$. Let S be the solid obtained by rotating the region R about the line $x = -3$.

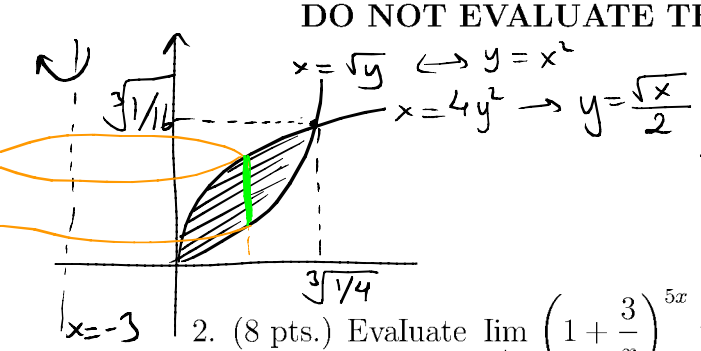
   (a) Write the integral for finding the volume of S using **the disk/washer method**. **DO NOT EVALUATE THIS INTEGRAL.**



$y = 16y^4$

$$V = \int_{y=0}^{y=\sqrt[3]{1/16}} \pi \left(\underbrace{(3+\sqrt{y})^2}_{\text{outer}} - \underbrace{(3+4y^2)^2}_{\text{inner}} \right) dy$$

   (b) Write the integral for finding the volume of S using **the shell method**. **DO NOT EVALUATE THIS INTEGRAL.**



$$V = \int_{x=0}^{x=\sqrt[3]{\frac{1}{4}}} 2\pi \underbrace{(3+x)}_{r} \cdot \underbrace{\left(\frac{\sqrt{x}}{2} - x^2\right)}_{h} dx$$

2. (8 pts.) Evaluate $\lim_{x\to\infty} \underbrace{\left(1+\frac{3}{x}\right)}_{y}^{5x}$ without using any formulae.

$$\lim_{x\to\infty} \ln y = \lim_{x\to\infty} 5x \cdot \ln\left(1+\frac{3}{x}\right) = \lim_{x\to\infty} \frac{5 \cdot \ln\left(1+\frac{3}{x}\right)}{1/x}$$

$$\underset{L'H.}{\overset{(\frac{0}{0})}{=}} \lim_{x\to\infty} \frac{5 \cdot \frac{1}{1+\frac{3}{x}} \cdot \left(0 - \frac{3}{x^2}\right)}{\frac{-1}{x^2}} = \lim_{x\to\infty} \frac{15}{1+\frac{3}{x}} = \frac{15}{1+0} = 15$$

$$\Rightarrow \lim_{x\to\infty} y = \lim_{x\to\infty} e^{\ln y} = e^{\lim_{x\to\infty} \ln y} = \boxed{e^{15}}$$

3. (8 pts.) Suppose f^{-1} is the inverse of a differentiable function f, $f(5) = 4$ and that $f'(5) = \frac{3}{2}$. Calculate $\left(f^{-1}\right)'(4)$.

$$\left(f^{-1}\right)'(4) = \frac{1}{f'\left(f^{-1}(4)\right)} = \frac{1}{f'(5)} = \frac{1}{3/2} = \frac{2}{3}$$

4. ($2 \times 6 = 12$ pts.) Find the following indefinite integrals.

(a) $\int \frac{y+3}{y^2+1} + \frac{4}{1-y}\, dy$

$$= \int \frac{y}{y^2+1}\, dy + 3\int \frac{dy}{y^2+1} + 4\cdot\int \frac{dy}{1-y}$$

$u = y^2+1$, $du = 2y\,dy$; $v = 1-y$, $dv = -dy$

$$= \int \frac{du/2}{u} + 3\arctan y - 4\int \frac{dv}{v} = \frac{\ln|u|}{2} + 3\arctan y - 4\ln|v| + C$$

$$= \frac{1}{2}\ln(1+y^2) + 3\arctan y - 4\ln|1-y| + C$$

(b) $\int x\, 3^{x^2+1}\, dx$

$$= \frac{1}{2}\int 3^u\, du = \frac{1}{2}\, 3^u \cdot \frac{1}{\ln 3} + C$$

$u = x^2+1$, $du = 2x\,dx$

$$= \frac{1}{2\ln 3}\, 3^{x^2+1} + C.$$

5. (10 pts.) Write **the form** of the Partial Fraction Decomposition that is needed to calculate the following integral. **DO NOT CALCULATE THE COEFFICIENTS.**

$$\int \frac{22x^2 + 5x + 2012}{(x+3)(x+1)(x-1)^2(5x^2+2x+10)(x^2+4)^2}\, dx$$

$$\int \left(\frac{A}{x+3} + \frac{B}{x+1} + \frac{C}{x-1} + \frac{D}{(x-1)^2} + \frac{Ex+F}{5x^2+2x+10} + \frac{Gx+H}{x^2+4} + \frac{Ix+J}{(x^2+4)^2} \right) dx$$