METU - NCC

	Precalculus Final	

Code : *Math 100* Acad.Year: *2014-2015* Semester : *Fall* Date : *14.1.2014* Time : *9:00* Duration : *120 minutes*	Last Name: Name : KEY Department: Signature: Student No.: Section: 10 QUESTIONS ON 4 PAGES TOTAL 100 POINTS

1 (8)	2 (10)	3 (8)	4 (8)	5 (10)	6 (12)	7 (12)	8 (10)	9 (10)	10 (12)		

1. *(8 pts)* Write the equations of the following lines through the point (6, 7).

(a) Which is perpendicular to the line $y = 3x + 5$.

Let m be the slope of the line. Since it is perpendicular to the line $y=3x+5$, $3 \cdot m = -1 \Rightarrow m = -\frac{1}{3}$

The line eqn. passing through (6,7) is: $\frac{y-7}{x-6} = -\frac{1}{3}$ OR $y = -\frac{x}{3} + 9$

(b) Which does not intersect the line $y = 2x + 1$.

Let m be the slope of the line. Since it is not intersecting the line $y = 3x+5$, it must be parallel, so $m = 2$.

The line eqn passing through (6,7) is: $\frac{y-7}{x-6} = 2$ OR $y = 2x - 5$

2. *(10 pts)* Solve each equation.

(a) $e^{2x} - 6e^x = -9$

Say $e^x = a$. Eqn becomes $a^2 - 6a + 9 = 0 \Rightarrow (a-3)^2 = 0 \Rightarrow a = 3$

It means, $e^x = 3$. By taking logarithm of both sides,

$\Rightarrow \ln e^x = \ln 3$

$\Rightarrow x = \ln 3$

(b) $\log_2(\frac{x^6}{2}) = (\log_2(x^3))^2$

$\Rightarrow \log_2(x^6) - \log_2 2 = (\log_2(x^3))^2$

$6 \log_2(x) - 1 = (3 \log_2(x))^2$

Say $\log_2(x) = a$ then eqn becomes;

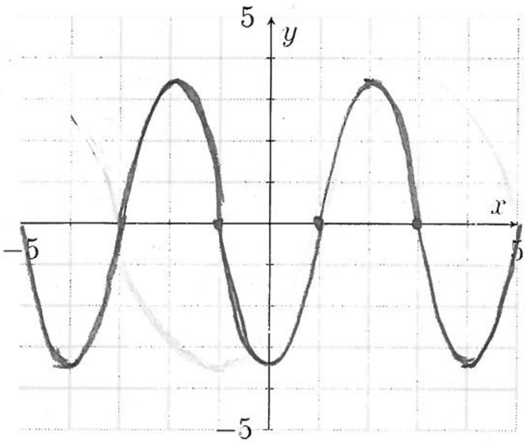
$\Rightarrow 6a - 1 = (3a)^2$

$\Rightarrow 9a^2 - 6a + 1 = 0$

$\Rightarrow (3a-1)^2 = 0$

$\Rightarrow a = \frac{1}{3}$ This means, $\log_2 x = \frac{1}{3} \Rightarrow x = 2^{1/3}$ OR $\sqrt[3]{2}$

3. *(8 pts)* Sketch the graph of $f(x) = A\sin(Bx + C)$ for $-5 \le x \le 5$ where the period of function $f(x)$ is 4, amplitude is 3.5 and it passes through the point $(1, 0)$.



4. *(8 pts)* Find the following values.

(a) $\cos(\frac{49\pi}{4}) = \cos\left(\frac{\pi}{4}\right)$

$= \frac{\sqrt{2}}{2}$

$\csc(\frac{-51\pi}{12}) = \csc\left(\frac{21\pi}{12}\right)$

$= \csc\left(\frac{7\pi}{4}\right)$

$= -\sqrt{2}$

(b) $\arctan(\sqrt{2}\sin(225°)) = \arctan\left(\sqrt{2} \cdot -\frac{\sqrt{2}}{2}\right)$

$= \arctan(-1)$

$= -\frac{\pi}{4}$

$\tan(-225°) = \tan(135°)$

$= -1$

5. *(10 pts)* Compute the following values by using related trigonometric formulas.

(a) $\sin(157.5°) = \sin(22.5°) = \sin\left(\frac{45°}{2}\right)$

$= \sqrt{\frac{1-\cos(45°)}{2}}$

$= \sqrt{\frac{1-\sqrt{2}/2}{2}}$ OR $\frac{\sqrt{2-\sqrt{2}}}{2}$

(b) $\cos(75°)\cos(15°) = \sin(15°) \cdot \cos(15°)$

$= \frac{1}{2} \cdot \sin(30°)$

$= \frac{1}{4}$

6. *(12 pts)* Prove the following trigonometric identity:

$$\frac{\sin 3x}{\sin 2x} = 2\cos x - \frac{1}{2\cos x}$$

$$\frac{\sin 3x}{\sin 2x} = \frac{\sin(2x+x)}{\sin 2x} \overset{\text{sum}}{=} \frac{\sin 2x \cos x + \cos 2x \sin x}{\sin 2x}$$

double angle

$$= \frac{2.\sin x.\cos x.\cos x + (2\cos^2 x - 1)\sin x}{2\sin x \cos x}$$

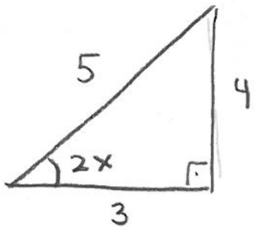
$$= \frac{2\sin x.\cos^2 x + 2\sin x \cos^2 x - \sin x}{2\sin x \cos x}$$

$$= \frac{4\sin x \cos^2 x - \sin x}{2\sin x \cos x}$$

$$= \frac{\cancel{4}^{2}\cancel{\sin x}\cos^{\cancel{2}} x}{\cancel{2}\cancel{\sin x}\cancel{\cos x}} - \frac{\cancel{\sin x}}{2\cancel{\sin x}\cos x}$$

$$\boxed{2\cos x - \frac{1}{2\cos x}}$$

7. *(12 pts)* If $\tan 2x = \dfrac{4}{3}$ and x is in the first quadrant then use basic trigonometric identities to find $\tan 3x$.



$$\tan 2x = \frac{4}{3} \Rightarrow \cos 2x = \frac{3}{5}$$

Using half-angle formula;

$$\tan x = \sqrt{\frac{1-\cos 2x}{1+\cos 2x}} = \sqrt{\frac{1-\frac{3}{5}}{1+\frac{3}{5}}} = \sqrt{\frac{\frac{2}{5}}{\frac{8}{5}}} = \frac{1}{2}$$

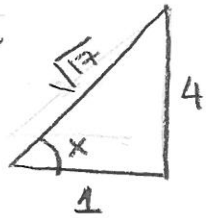
$$\tan 3x = \tan(2x+x) = \frac{\tan 2x + \tan x}{1 - \tan 2x \tan x}$$

$$= \frac{\frac{4}{3} + \frac{1}{2}}{1 - \frac{4}{3}.\frac{1}{2}}$$

$$= \frac{\frac{11}{6}}{\frac{2}{6}} \Rightarrow \tan 3x = \frac{11}{2}$$

8. *(10 pts)* Find $\tan y$ if we know that $0 < x < \pi$, $\sec x = \sqrt{17}$ and $\tan(x - y) = 2$

We have



$\tan x = 4$ and $\tan(x-y) = 2$.

$$\tan y = \tan(x - (x-y)) = \frac{\tan x - \tan(x-y)}{1 + \tan x \tan(x-y)}$$

$$\tan y = \frac{4-2}{1+4\cdot 2} = \frac{2}{9}$$

9. *(10 pts)* Determine whether the following triangles are possible or impossible by using law of sine and/or law of cosine.

(a) Sides of the triangle ABC: $a = 5$, $b = 13$ and angle: $\hat{A} = 60°$, $\hat{B} = 90°$

If there is such triangle, then it should satisfy Law of Sine:

$$\frac{a}{\sin A} = \frac{b}{\sin B} \Rightarrow \frac{5}{\sin 60°} = \frac{13}{\sin 90°}$$

$$\Rightarrow \frac{5}{\frac{\sqrt{3}}{2}} \neq \frac{13}{1}$$ So, it is impossible.

(b) Sides of the triangle ABC: $a = 7$, $b = 3$, $c = 5$ and the angle: $\hat{A} = 120°$.

If there is such triangle, then it should satisfy Law of Cosine:

$$a^2 = b^2 + c^2 - 2bc\,\text{Cos}\,A \Rightarrow 7^2 = 3^2 + 5^2 - 2\cdot 3\cdot 5\cdot \text{Cos}\,120°$$

$$49 = 9 + 25 - \cancel{2}\cdot 3\cdot 5\cdot\left(-\frac{\sqrt{3}}{\cancel{2}}\right)$$

$$49 = 34 + 15\sqrt{3}$$

$15 \neq 15\sqrt{3}$ so, it is impossible.

10. *(12 pts)* Solve the following trigonometric equation for $x \in [0, 2\pi]$:

$$\sin^2(x) = \frac{\sqrt{3}}{2}\sin(2x)$$

$$\sin^2 x = \frac{\sqrt{3}}{\cancel{2}}\cancel{2}\cdot\sin x\cdot\text{Cos}\,x \Rightarrow \sin x\,(\sin x - \sqrt{3}\,\text{Cos}\,x) = 0$$

$$\sin x = 0 \Rightarrow x = 0, \pi, 2\pi$$

$$\sin x - \sqrt{3}\,\text{Cos}\,x = 0 \Rightarrow \tan x = \sqrt{3} \Rightarrow x = \frac{\pi}{3}, \frac{4\pi}{3}$$

$$\boxed{x = 0, \frac{\pi}{3}, \frac{4\pi}{3}, \pi, 2\pi}$$