M E T U
Northern Cyprus Campus

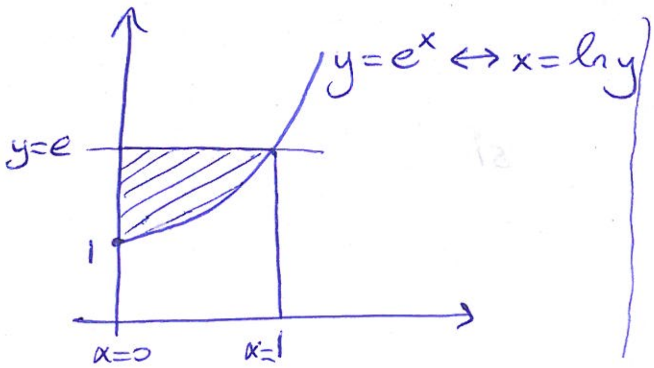
	Calculus for Functions of Several Variables Short Exam 2	
Code : *Math 120* Acad. Year: *2014-2015* Semester : *Summer* Date : *04.08.2015* Time : *18:20* Duration : *30 minutes*	Last Name: Name: Signature:	List No: Student No:
	4 QUESTIONS ON 2 PAGES TOTAL 20 POINTS	

1(4)	2(6)	3(8)	4(2)			

Show your work! No calculators! Please draw a $\boxed{\text{box}}$ around your answers!
Please do not write on your desk!

1. (4 pts.) Evaluate the following integral by changing the order of integration.

$$\int_0^1 \int_{e^x}^{e} \frac{y}{\ln(y)}\, dy\, dx \quad = \int_{y=1}^{y=e} \int_{x=0}^{x=\ln y} \frac{y}{\ln y}\, dx\, dy$$

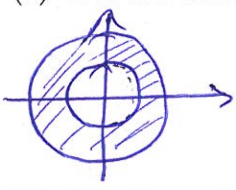


$$= \int_{y=1}^{y=e} y\, dy = \left.\frac{y^2}{2}\right|_{y=1}^{y=e}$$

$$= \boxed{\frac{e^2-1}{2}}$$

2. ($3 \times 2 = 6$ pts.) In the following questions, write the region R in **polar coordinates.**

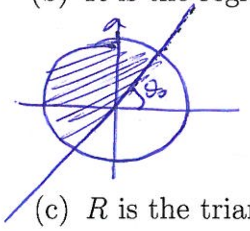
(a) R is the annulus (washer) between the circles $x^2 + y^2 = 3$ and $x^2 + y^2 = 5$.



$0 \leq \theta \leq 2\pi$

$\sqrt{3} \leq r \leq \sqrt{5}$

(b) R is the region inside the circle $x^2 + y^2 = 4$ that is above the line $y = 2x$.

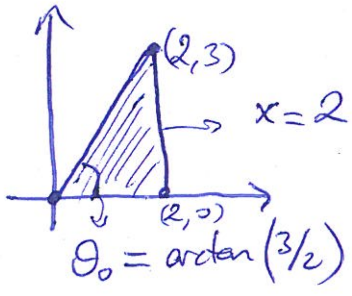


$\theta_0 = \arctan 2$

$\arctan 2 \leq \theta \leq \pi + \arctan 2$

$0 \leq r \leq 2$

(c) R is the triangle with vertices $(0,0)$, $(2,0)$ and $(2,3)$.



$x = 2 \Leftrightarrow r\cos\theta = 2$
$r = 2\sec\theta$

$0 \leq \theta \leq \arctan(3/2)$

$0 \leq r \leq 2\sec(\theta)$

3. $(4+4=8 \text{ pts.})$

(a) Show that the vector field $F(x,y) = \left\langle \underbrace{2xy^3\cos(x^2)}_{P}, \underbrace{3y^2\sin(x^2)+1}_{Q} \right\rangle$ is *conservative*.

① Dom $F = \mathbb{R}^2$ obviously

② $P_y = 2x \cdot 3y^2 \cdot \cos(x^2) = 6xy^2\cos(x^2)$

$Q_x = 3y^2\cos(x^2) \cdot 2x + 0 = 6xy^2\cos(x^2)$

So F is conservative.

(b) Find *a potential function* $\phi(x,y)$ for the $F(x,y)$ above.

$\phi_x = 2xy^3\cos(x^2)$

$\phi_y = 3y^2\sin(x^2)+1$

$\phi = \int \phi_y\, dy = \int (3y^2\sin(x^2)+1)\, dy$

$\boxed{\phi = y^3\sin(x^2) + y + g(x)}$

$2xy^3\cos(x^2) = \phi_x = 2xy^3\cos(x^2) + 0 + g'(x)$

$g'(x) = 0 \Rightarrow g(x) = K.$

$\boxed{\phi = y^3\sin(x^2) + y + K}$

4. (2 pts.) Evaluate $\int_C \left\langle 2xy^2+1\,,\, 2x^2y+2y \right\rangle \bullet dr$ where C is the quarter-circle $\{x^2+y^2=4, x \geq 0, y \geq 0\}$, starting from the point $(2,0)$ and ending at the point $(0,2)$ **by using Fundamental Theorem of Line Integrals only.**

Other methods will not receive any credit.

Notice that $\phi(x,y) = x^2y^2 + x + y^2$ is *a potential function* for the vector field above.

$\int_C \langle 2xy^2+1, 2x^2y+2y \rangle \cdot dr$

$= \phi(0,2) - \phi(2,0) = 2^2 - 2$

$= \boxed{2}$