M E T U
Northern Cyprus Campus

	Calculus for Functions of Several Variables Short Exam 1	
Code : *Math 120* Acad.Year: *2014-2015* Semester : *Summer* Date : *10.07.2015* Time : *12:40* Duration : *20 minutes*	Last Name: Name: KEY Signature: Student No:	
	2 QUESTIONS 2 PAGES TOTAL 10 POINTS	

1(5)	2(5)			

Show your work! No calculators! Please draw a $\boxed{\text{box}}$ around your answers!
Please do not write on your desk!

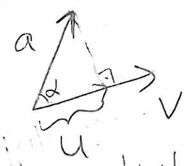
1. ($5 \times 1 = 5$ pts.) Let P be the point $(5, 6, 7)$ and the line L with parametric equations $x(t) = 1 + 3t$, $y(t) = t - 2$, $z(t) = 3$. Notice that the point $Q = (1, -2, 3)$ is on this line.

(a) Find the vector $\mathbf{a} = \vec{QP}$. $= \langle 5-1, 6+2, 7-3 \rangle = \langle 4, 8, 4 \rangle$.

(b) Find a **direction vector**, $\mathbf{v}$ for the line L. $\langle 3, 1, 0 \rangle$.

(c) Find the vector projection of $\mathbf{a}$ onto $\mathbf{v}$, i.e., $\boxed{\text{Proj}_{\mathbf{v}}\, \mathbf{a}. = u}$

$$\frac{(a.v)}{|v|}\frac{v}{|v|} = \frac{(12+8+0)}{10}\langle 3, 1, 0 \rangle = 2v = \langle 6, 2, 0 \rangle.$$



$$|u| = |a|\cos\alpha = \frac{|a|.|v|.\cos\alpha}{|v|} = \frac{a.v}{|v|}$$

(d) Find the projection of $\mathbf{a}$ orthogonal to $\mathbf{v}$, i.e., $\text{Proj}^{\perp}_{\mathbf{v}}\, \mathbf{a}$. $= a - \text{proj}_v a$.

$$= \langle 4, 8, 4 \rangle - \langle 6, 2, 0 \rangle = \langle -2, 6, 4 \rangle$$

(e) Find the length of $\text{Proj}^{\perp}_{\mathbf{v}}\, \mathbf{a}$.

$$\sqrt{4 + 36 + 16} = \sqrt{56}$$

Congratulations : You just found the distance of the point P to the line L.

2. (3+2 = 5 pts.) Consider the following pair of lines that are given via parametric equations.

$$L_1 : x = 2t + 10 \qquad y = 7 - t \qquad z = 3t + 2015$$

$$L_2 : x = 10 - s \qquad y = 5s + 7 \qquad z = 2015$$

Notice that these lines intersect at a point that is obviously visible from the equations.

(a) Find a vector that is perpendicular to both of these lines.

$u_1 = \langle 2, -1, 3 \rangle$, $u_2 = \langle -1, 5; 0 \rangle$.

$$u_1 \times u_2 = \begin{vmatrix} i & j & k \\ 2 & -1 & 3 \\ -1 & 5 & 0 \end{vmatrix} = i(-15) - j(3) + k(10-1)$$

$$= \langle -15, -3, 9 \rangle.$$

(b) Write any equation of the line that is perpendicular to both of these lines and passes from their intersection point.

intersection point $= (10, 7, 2015)$ $(s = t = 0)$.

$$\boxed{\begin{aligned} x &= 10 + (-15)t \\ y &= 7 + (-3)t \\ z &= 2015 + 9t \end{aligned}} \quad \text{OR} \quad \boxed{(10, 7, 2015) + t(-15, -3, 9)}$$

Congratulations!
You have found the normal line to the unique plane that contains L_1 and L_2.

DID YOU WRITE YOUR NAME AND STUDENT NUMBER?