M E T U
Northern Cyprus Campus

	Calculus for Functions of Several Variables Short Exam	
Code : *Math 120* Acad.Year: *2014* Semester : *Summer* Date : *15.7.2014* Time : *17:00* Duration : *45 minutes*	Last Name: Name: KEY Student No: Signature: 7 QUESTIONS ON 4 PAGES TOTAL 30+2=32 POINTS	

1(4)	2(4)	3(4)	4(4)	5(4)	6(4)	7(8)			

Show your work! No calculators! Please draw a $\boxed{\text{box}}$ around your answers!
Please do not write on your desk!

1. $(2+2=4$ pts.)

   (a) Write an equation for the line that is parallel to the vector $a = \langle 1, -1, 2\rangle$ which passes from the point $(1, 2, 3)$.

   $x(t) = 1+t \quad , y(t) = 2-t \quad , z(t) = 3+2t \qquad t \in \mathbb{R}$

   (b) Find the **point** where the line above intersects the plane $2x + 3y + z = 11$.

   $2(1+t) + 3(2-t) + (3+2t) = 11$

   $\underbrace{2t - 3t + 2t}_{t} + 2 + 6 + 3 = 11$

   $= 11 - 11 = 0$

   $t = 0 \Rightarrow \quad x(0) = 1, \quad y(0) = 2, \quad z(0) = 3$

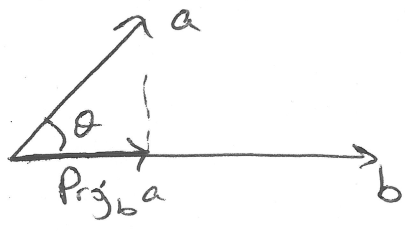
   $(1,2,3)$ is the point of intersection.

2. $(2+2=4$ pts.) Let $a = \langle 1, -1, 2\rangle$ and $b = \langle 1, -1, 2\rangle$.

   (a) Find the vector projection of a onto b, i.e., $\text{Proj}_{\mathbf{b}}\, \mathbf{a}$.

   $\text{Proj}_b\, a = \|a\| \cdot \cos\theta \cdot \frac{b}{\|b\|} = \frac{\|a\| \cdot \|b\| \cdot \cos\theta}{\|b\|^2} \cdot b$

   $= \frac{1\cdot 1 + (-1)\cdot(-1) + 2\cdot 2}{1\cdot 1 + (-1)(-1) + 2\cdot 2} \cdot b = 1 \cdot b = b$



   (b) Find the projection of a orthogonal b, i.e., $\text{Proj}^{\perp}_{\mathbf{b}}\, \mathbf{a}$.

   $\text{Proj}^{\perp}_b\, a = b - \text{Proj}_b\, a = b - b = 0.$

   (This makes sense because $a = b$)

3. (2 + 2 = 4 pts.) This question has two unrelated parts.

(a) Find an equation of the plane perpendicular to $\langle 1, -1, 2\rangle$ passing from the point $(1, 1, 0)$.

$$x - y + 2z = 1 - 1 + 2\cdot 0 = 0$$

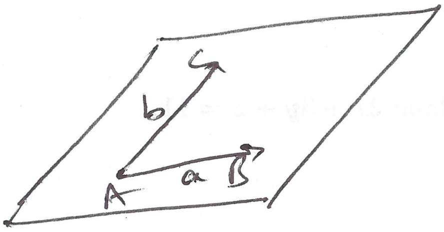
(b) Find an equation of the plane which passes through the origin and is parallel to the plane $100x + 119y + 120z = 219$.

parallel $\Rightarrow$ $100x + 119y + 120z = d$

has to satisfy $(0,0,0) \Rightarrow d = 100\cdot 0 + 119\cdot 0 + 120\cdot 0$

$\Rightarrow d = 0$ $\qquad 100x + 119y + 120z = 0$

4. (4 pts.) Find an equation of the plane which passes through the points $A(1, 2, 3)$, $B(-2, 3, 3)$, and $C(1, 3, 4)$.



$$a = \vec{AB} = \langle -3, 1, 0\rangle$$

$$b = \vec{AC} = \langle 0, 1, 1\rangle$$

$$n = a \times b = \begin{vmatrix} i & j & k \\ -3 & 1 & 0 \\ 0 & 1 & 1 \end{vmatrix} = \langle 1\cdot 1 - 1\cdot 0, 0\cdot 1 - (-3\cdot 1), -3\cdot 1 - 0\cdot 1\rangle$$

$$= \langle 1, 3, -3\rangle$$

Π : $\qquad x + 3y - 3z = d$

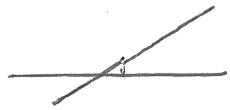
Plug in A : $\qquad 1 + 3\cdot 2 - 3\cdot 3 = d$

$\Rightarrow d = -2$

Π : $\qquad x + 3y - 3z = -2$

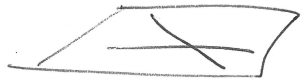
5. $(4 \times 1 = 4$ pts.) Determine whether the following statements in Cartesian 3−space are true or false. Indicate your answers with the words **TRUE** or **FALSE** to the left of the question. No explanations required.

- FALSE • Two lines either intersect or are parallel.



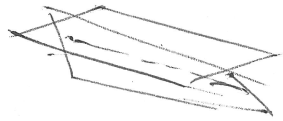
SKEW

- FALSE • Two lines parallel to a plane are parallel.



- TRUE • Two planes either intersect or are parallel.

- FALSE • Two planes parallel to a line are parallel.



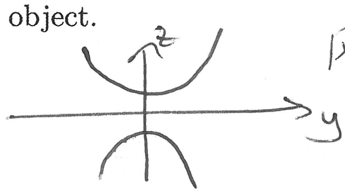
"pencil of planes"

6. $(4 \times 1 = 4$ pts.) Consider the level surface $x^2 + 3y^2 - 2z^2 = -5$.

- Sketch the slice for $x = 0$ and name the object.

$3y^2 - 2z^2 = -5$

hyperbola

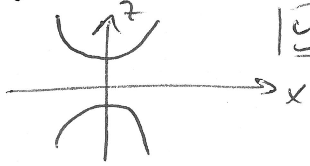


$\boxed{x=0}$

- Sketch the slice for $y = 0$ and name the object.

$x^2 - 2z^2 = -5$

hyperbola

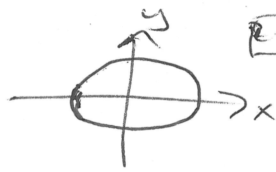


$\boxed{y=0}$

- Sketch the slice for $z = 2$ and name the object.

$x^2 + 3y^2 - 8 = -5$ or $x^2 + 3y^2 = 3$

ellipse



$\boxed{z=2}$

- What is the full technical name for the quadric $x^2 + 3y^2 - 2z^2 = -5$?

Notice $z = 0 \Rightarrow x^2 + 3y^2 = -5$ no soln

$\Rightarrow$ object is in two pieces.

So it is an

elliptic hyperboloid of two sheets.

7. ($2 \times 4 = 8$ pts.) Find the limit, if it exists and prove your claim. Otherwise, show that the limit does not exist.

(a) $\displaystyle\lim_{(x,y)\to(1,2)} \frac{(x-1)}{(x-1)^2+(y-2)^2}$

On $y=2$ $\quad \displaystyle\lim_{x\to 1} \frac{x-1}{(x-1)^2} = \lim_{x\to 1} \frac{1}{(x-1)}$ DNE.

Therefore the limit DNE.

(b) $\displaystyle\lim_{(x,y)\to(0,0)} \frac{x^3y}{x^6+y^3}$

On $y = kx^2$, $k \neq 0$, $\quad \displaystyle\lim_{x\to 0} \frac{kx^5}{2x^6} = \lim_{x\to 0} \frac{k}{2 \cdot x}$ DNE

So lim DNE.

DID YOU KNOW YOU WOULD GET A 0 IF YOU DID NOT WRITE YOUR NAME?