M E T U
Northern Cyprus Campus

	Calculus for Functions of Several Variables **Short Exam 1**	

Code : *Math 120* Acad.Year: *2012-2013* Semester : *Summer* Date : *11.7.2012* Time : *17:45* Duration : *35 minutes*	Last Name: Department: Section: Recitation:	Name: Student No: Signature:
	4 QUESTIONS ON 2 PAGES TOTAL 20 POINTS	

1	2	3	4					
						KEY		

Show your work! No calculators! Please draw a $\boxed{\text{box}}$ around your answers!
Please do not write on your desk!

1. ($5 \times 1 = 5$ pts.) Write an equation that describes each of the following sets inside the Cartesian 3−space.

   (a) The z−axis.
   $\{x=0, y=0\}$ or $r(t) = \langle 0, 0, t\rangle \quad t \in \mathbb{R}$

   (b) The yz−plane.
   $x=0$

   (c) A normal vector to the yz−plane.
   $\langle 1, 0, 0\rangle$

   (d) The line with direction vector $\mathbf{v} = (0, 0, 1)$ that passes from the point $(0, 0, 0)$.
   $r(t) = \langle 0, 0, t\rangle \quad t \in \mathbb{R}$

   (e) The plane with normal vector $\mathbf{n} = (1, 0, 0)$ that passes from the point $(0, 11, 7)$.
   $x = 0$

2. (2 pts.) Let $\mathbf{a} = \langle -4, 10, 2\rangle$ and $\mathbf{b} = \langle 1, 11, 7\rangle$

   (a) The **scalar projection** of $\mathbf{b}$ onto $\mathbf{a} = \text{Comp}_{\mathbf{a}}\mathbf{b} =$
   $$\frac{a \cdot b}{\|a\|} = \frac{-4\cdot 1 + 10\cdot 11 + 2\cdot 7}{\sqrt{(-4)^2 + 10^2 + 2^2}} = \frac{120}{\sqrt{120}} = \sqrt{120}$$

   (b) The **(vector) projection** of $\mathbf{b}$ onto $\mathbf{a} = \text{Proj}_{\mathbf{a}}\mathbf{b} =$
   $$\frac{a \cdot b}{\|a\|^2} \cdot a = \frac{\sqrt{120}}{\sqrt{120}} a = a = \langle -4, 10, 2\rangle$$

3. ($3 \times 2 = 6$ pts.) Identify the following surfaces as an *elliptical paraboloid, hyperbolic paraboloid, a hyperboloid of one sheet, a hyperboloid of two sheets, a cone, a circular cylinder, an elliptical cylinder, or a parabolic cylinder*, and identify the axis of symmetry as the x-axis, the y-axis, or the z-axis.

   (a) $2z^2 - 9x^2 + 5y^2 + 1 = 0 \iff 9x^2 - 5y^2 - 2z^2 = 1$ hyp. of 2 sheets ; x-axis

   (b) $6x^2 - 3y^2 - z^2 = 0 \iff 6x^2 = 3y^2 + z^2$ cone ; x-axis

   (c) $3y^2 + 3z^2 = 5 \iff$ circular cylinder ; x-axis

4. (3 pts.) Determine whether the given lines are parallel, intersecting, or skew. If they intersect, find the intersection point. Show your work.

$$L_1 : x = t,\ y = 7,\ z = 2t + 98$$
$$L_2 : x = -2s + 25,\ y = s,\ z = s + 113$$

$V_1 = \langle 1, 0, 2\rangle$
$V_2 = \langle -2, 1, 1\rangle$

$\begin{cases} 1 = -2t \\ 0 = t \\ 2 = t \end{cases}$ has no soln so $V_1 \nparallel V_2 \Rightarrow L_1 \nparallel L_2$

Intersection ?

$\left.\begin{array}{l} t = -2s + 25 \\ 7 = s \\ 2t + 98 = s + 113 \end{array}\right\}$ $\Rightarrow s = 7 \Rightarrow t = 11$

3rd eqn $2 \cdot 11 + 98 = 7 + 113$ ✓

So $s = 7$, $t = 11$ solves the system to give us the only intersection point $(11, 7, 120)$

$L_1 \cap L_2 = \{(11, 7, 120)\}$

5. (4 pts.) Find an equation of the plane that passes through the points $P = (1, 2, 3)$, $Q = (2, 2, 4)$, and $R(1, 2, -3)$

$\vec{PQ} = \langle 1, 0, 1\rangle$ & $\vec{PR} = \langle 0, 0, -6\rangle$ are on the plane

$$\vec{n} = \begin{vmatrix} i & j & k \\ 1 & 0 & 1 \\ 0 & 0 & -6 \end{vmatrix} = \langle 0, 6, 0\rangle$$

$6y = 12$ or $\boxed{y = 2}$

2nd solution: Notice all of P, Q, R are on $y = 2$ since their y-coordinates are all 2.

So $y = 2$ is the unique plane containing P, Q, R.