M E T U
Northern Cyprus Campus

	Calculus for Functions of Several Variables Short Exam 1	
Code : *Math 120* Acad.Year: *2013-2014* Semester : *Fall* Date : *28.10.2013* Time : *17:45* Duration : *35 minutes*	Last Name: Name: Student No: Signature:	
	4+1 QUESTIONS ON 2 PAGES TOTAL 20+2=22 POINTS	

1(4)	2(6)	3(5)	4(5)	B(2)	KEY		

Show your work! No calculators! Please draw a box around your answers!
Please do not write on your desk!

1. $(4 \times 1 = 4$ pts.) Let $\mathbf{a} = \langle 1, 2, -3\rangle$ and $\mathbf{b} = \langle 5, -3, 0\rangle$.

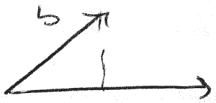
   (a) The line with direction vector $\mathbf{a}$ that passes from the point $(28, 10, 2013)$.

   $r(t) = \langle 1, 2, -3\rangle t + \langle 28, 10, 2013\rangle$

   (b) The plane with normal vector $\mathbf{b}$ that passes from the point $(28, 10, 2013)$.

   $5x - 3y + 0z = 110$ or $\langle 5, -3, 0\rangle \cdot \langle x-28, y-10, z-2013\rangle =$

   (c) Find **the vector projection** of the vector $\mathbf{b}$ onto the vector $\mathbf{a}$.

   $$\text{Proj}_{\mathbf{a}}\mathbf{b} = \|\mathbf{b}\| \cdot \cos\theta \cdot \frac{\mathbf{a}}{\|\mathbf{a}\|} = \frac{\mathbf{a}\cdot\mathbf{b}}{\|\mathbf{a}\|^2} = \frac{1\cdot 5 + 2\cdot(-3) + (-3)\cdot 0}{\sqrt{14}^2}\cdot \mathbf{a}$$
   $$= \frac{-5}{14}\langle 1, 2, -3$$

   (d) Find **the orthogonal projection** of the vector $\mathbf{b}$ onto the vector $\mathbf{a}$, that is the component of $\mathbf{b}$ perpendicular to $\mathbf{a}$

   $$\text{Proj}_{\mathbf{a}}^{\perp}\mathbf{b} = \mathbf{b} - \text{Proj}_{\mathbf{a}}\mathbf{b} = \langle 5, -3, 0\rangle + \left\langle \frac{5}{14}, \frac{10}{14}, \frac{-15}{14}\right\rangle$$
   $$= \left\langle \frac{75}{14}, \frac{-32}{14}, \frac{-15}{14}\right\rangle$$

2. $(3 \times 2 = 6$ pts.) **Identify** the following surfaces as an *elliptical paraboloid, hyperbolic paraboloid, a hyperboloid of one sheet, a hyperboloid of two sheets, a cone, a circular cylinder, an elliptical cylinder, or a parabolic cylinder.*
   Identify the axis of symmetry as the x-axis, the y-axis, or the z-axis.

   (a) $3x^2 - y^2 - z^2 + 1 = 0 \iff -3x^2 + y^2 + z^2 = +1$
   (circular) hyperboloid of one sheet ; x-axis

   (b) $x^2 + 5y - 3z^2 + 1 = 0 \iff 5y + 1 = 3z^2 - x^2$
   (saddle) or hyperbolic paraboloid ; y-axis.

   (c) $x^2 + 2y^2 - 3z^2 = 0 \iff 3z^2 = x^2 + 2y^2$
   elliptic cone ; z-axis

3. (5 pts.) Determine whether the given lines are parallel, intersecting, or skew. If they intersect, find the intersection point. Show your work.

$L_1 : x = t,\ y = 2t - 46,\ z = 3t + 1929$

$L_2 : x = 28,\ y = 5s + 610,\ z = 813 - 10s$

$v_1 = \langle 1, 2, 3 \rangle$

$v_2 = \langle 0, 5, -10 \rangle$

Clearly not parallel

$$\begin{cases} t = 28 \\ 2t - 46 = 5s + 610 \\ 3t + 1929 = 813 - 10s \end{cases} \Rightarrow t = 28 \Rightarrow 2 \cdot 28 - 46 = 5s + 610$$

$5s = -600 \Rightarrow s = -120$

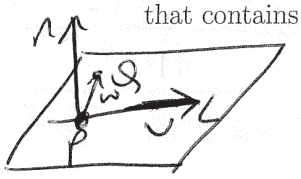
3rd eqn.? $3 \cdot 28 + 1929 \stackrel{?}{=} 813 - 10(-120)$

$2013 = 2013$ ✓

So these lines intersect, put $t = 28$ in L_1 to find the point $(28, 10, 2013)$

$L_1 \cap L_2 = \{(28, 10, 2013)\}$

4. (5 pts.) Find an equation of the plane that passes through the point $P = (1, 2, 3)$ and that contains the line $L : x = 2t + 8,\ y = 10t,\ z = 3$.



$v = \langle 2, 10, 0 \rangle$

$P = (8, 0, 3)$

$Q = (1, 2, 3)$

Let $w = \vec{PQ} = \langle -7, 2, 0 \rangle$

Then choose $\vec{n} = v \times w = \begin{vmatrix} i & j & k \\ 2 & 10 & 0 \\ -7 & 2 & 0 \end{vmatrix} = \langle 0, 0, 4 + 70 \rangle$

$= \langle 0, 0, 74 \rangle$ or easier $\vec{n} = \langle 0, 0, 1 \rangle$

Then Π: $\langle 0, 0, 1 \rangle \cdot \langle x - 8, y - 2, z - 3 \rangle = 0$

$z = 3$

BONUS solution: P & L are on the plane $z = 3$ (Look!)

hence $z = 3$ is the plane we want

5. Bonus $(1 + 1 = 2$ pts.) Determine whether the given statement is true or false. No explanations required.

F (a) In Cartesian 3−space, 3 points always determine a plane.

F (b) The primary online communication form in this course is **NOT** the announcements tab on http://www.math.ncc.metu.edu.tr/content/courses/math120/ , but e-mail instead.