M E T U
Northern Cyprus Campus

	Calculus for Functions of Several Variables Short Exam 1	
Code : *Math 120* Acad.Year: *2011-2012* Semester : *Spring* Date : *05.3.2012* Time : *17:45* Duration : *45 minutes*	Last Name: Department: Section: Recitation:	Name: Student No: Signature:
	7 QUESTIONS ON 4 PAGES TOTAL 50 POINTS	

1	2	3	4	5	6	7	

Show your work! No calculators! Please draw a $\boxed{\text{box}}$ around your answers!
Please do not write on your desk!

1. (4 pts.) Find the **center** and the **radius** of the sphere $x^2 + 2x + y^2 - 6y + z^2 + z = 21$.

$$(x^2+2x+1-1)+(y^2-6y+9-9)+(z^2+z+1/4-1/4) = 21$$
$$(x+1)^2+(y-3)^2+(z+1/2)^2-1-9-\frac{1}{4} = 21$$
$$(x+1)^2+(y-3)^2+\left(z+\frac{1}{2}\right)^2 = 31.25$$

Center : $(-1, 3, -1/2)$ Radius $= \sqrt{31.25}$

2. $(2 + 2 = 4$ pts.) Write an equation that describes each of the following sets.

(a) The y—axis. $r(t) = (0, t, 0) \quad t \in \mathbb{R}$

(b) The yz—plane. $x = 0$

3. $(3 + 3 = 6$ pts.) Write an equation that describes each of the following sets.

(a) The line with direction vector $\mathbf{v} = (1, 0, -2)$ that passes from the point $(3, 5, 7)$.

$$r(t) = t(1,0,-2)+(3,5,7) = (t+3, 5t, 7-2t) \quad t \in \mathbb{R}$$

(b) The plane with normal vector $\mathbf{n} = (2, 1, -3)$ that passes from the point $(4, 5, 6)$.

$$(2,1,-3)\cdot(x-4, y-5, z-6) = 0 \quad \text{OR} \quad 2x+y-3z = -5$$

4. $(3 + 3 = 6$ pts.) Let $\mathbf{a} = (-3, 0, 4)$ and $\mathbf{b} = (3, 2\sqrt{6}, 4)$

(a) The **scalar projection** of $\mathbf{b}$ onto $\mathbf{a} = \text{Comp}_{\mathbf{a}}\mathbf{b} =$ $\frac{a \cdot b}{|a|}$

$$\text{Comp}_a b = \frac{(-3)\cdot 3 + 0 \cdot 2\sqrt{6} + 4 \cdot 4}{\sqrt{(-3)^2 + 0^2 + 4^2}} = \frac{7}{5}$$

(b) The **(vector) projection** of $\mathbf{b}$ onto $\mathbf{a} = \text{Proj}_{\mathbf{a}}\mathbf{b} =$ $\frac{a \cdot b}{|a|}\frac{a}{|a|} = \frac{a \cdot b}{|a|^2} a = \frac{a \cdot b}{a \cdot a} a$

$$\text{Proj}_a b = \frac{7}{5}\frac{1}{5}(-3,0,4) = \frac{7}{25}(-3,0,4) = \left(\frac{-21}{25}, 0, \frac{28}{25}\right)$$

5. (10 pts.) Determine whether the given lines are parallel, intersecting, or skew. If they intersect, find the intersection point. Show your work.

$$L_1 : x = 2t + 1,\ y = -3t,\ z = 7t - 1$$

$$L_2 : x = s + 2,\ y = -s + 1,\ z = 3s$$

Direction vector of $L_1 = v_1 = (2, -3, 7)$

Direction vector of $L_2 = v_2 = (1, -1, 3)$

<u>v_1 is not parallel to v_2:</u> Assume $v_1 = k v_2$ for some $k \neq 0$

Then $2 = k$ and $-3 = -k$ which is a contradiction

Hence L_1 is not parallel to L_2.

If these lines were to intersect, they would intersect at a single point (since they are not parallel).

For that point:

$$\left.\begin{aligned} 2t+1 &= x = s+2 \\ -3t &= y = -s+1 \\ 7t-1 &= z = 3s \end{aligned}\right\}$$

2^{nd} eqn. gives $s = 3t+1$, then 1^{st} eqn. gives $2t+1 = 3t+3$

$\Rightarrow t = -2$ & $s = -5$

When we put these values in the third equation we see that $7(-2) - 1 = -15 = 3\cdot(-5)$, hence the system is consistent, i.e, there is an intersection point.

$t = -2$ on L_1 gives $(-3, 6, -15)$

$s = -5$ on L_2 gives $(-3, 6, -15)$

Hence $\{(-3, 6, -15)\} = L_1 \cap L_2$

6. (8 pts.) Find an equation of the plane consisting of all points that are equidistant from the points $(1, -4, -1)$ and $(0, 0, 2)$.

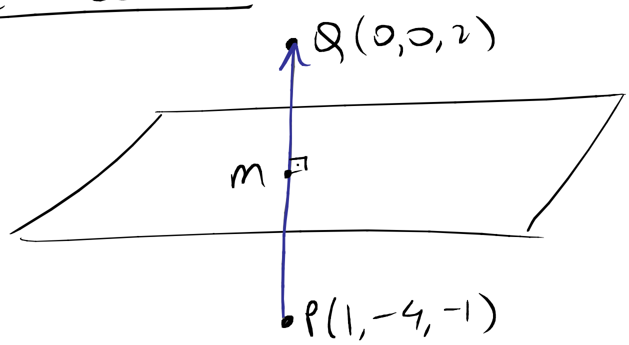
1^{st} solution: Say (x,y,z) is a point on this plane.

Then $\sqrt{(x-1)^2+(y+4)^2+(z+1)^2} = \sqrt{(x-0)^2+(y-0)^2+(z-2)^2}$

$$x^2-2x+1+y^2+8y+16+z^2+2z+1 = x^2+y^2+z^2-4z+4$$
$$-2x+8y+6z = -1-16-1+4$$
$$-2x+8y+6z = -14$$
$$x-4y-3z = 7$$

is the equation of the required plane.

2^{nd} Solution:

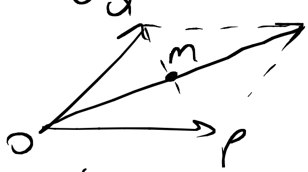


The vector $\vec{PQ} = (-1,4,3)$ serves as a normal, call it $\hat{n}$.

We need to find a point on the plane.

M = the midpoint of the line segment $\overline{PQ}$ is on the plane.

Finding M: 1^{st} method:



M is the endpoint of half of the vector $\vec{OP}+\vec{OQ}$

$$\vec{OM} = \frac{1}{2}\left((1,-4,-1)+(0,0,2)\right) = \left(\tfrac{1}{2},-2,\tfrac{1}{2}\right)$$

Then $M = \left(\tfrac{1}{2},-2,\tfrac{1}{2}\right)$

2^{nd} method:

$r(t) = t\cdot(-1,4,3)+(1,-4,-1) \quad t\in[0,1]$

is the parametrization of the line segment $\overline{PQ}$.

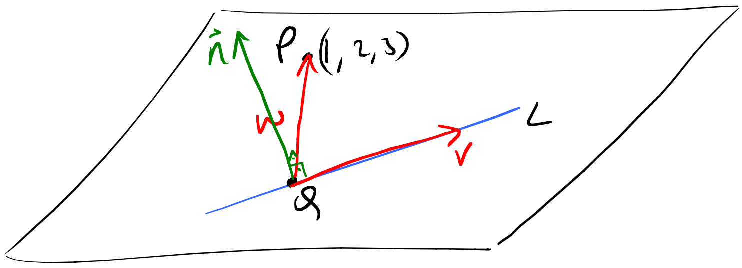
$t=\tfrac{1}{2}$ gives the midpoint $M = \left(\tfrac{1}{2},-2,\tfrac{1}{2}\right)$

Then plane eqn is

$(-1,4,3)\cdot\left(x-\tfrac{1}{2},y+2,z-\tfrac{1}{2}\right)=0 \iff -x+\tfrac{1}{2}+4y-8+3z-\tfrac{3}{2}=0$

$\iff -x+4y+3z=-7 \iff \boxed{x-4y-3z=7}$

7. (12 pts.) Find an equation of the plane that passes through the point $(1, 2, 3)$ that includes the line $x = 2t, \quad y = 1 - t. \quad z = 3t - 1.$



Direction vector of $L = (2, -1, 3)$

We need another vector on the plane Π.

Let $Q = (0, 1, -1) \in L \subset \Pi$

↑ obtained by plugging in $t=0$ in the eqn. of L)

Then $w = \vec{QP} = (1, 1, 4)$ is another vector inside Π.

Then $\vec{n} = v \times w$ will be a normal for the plane.

$$\vec{n} = v \times w = \begin{vmatrix} i & j & k \\ 2 & -1 & -3 \\ 1 & 1 & 4 \end{vmatrix} = \left((-1)4 - (1)(-3)\right)\vec{i} - \left(2\cdot 4 - 1\cdot(-3)\right)\vec{j} + \left(2\cdot 1 - 1\cdot(-1)\right)\vec{k}$$

$$\vec{n} = (-4+3, -(8+3), 2+1) = (-1, -11, 3)$$

Equation for Π :

$$(-1, -11, 3) \cdot (x-1, y-2, z-3) = 0$$

$$-x+1 - 11y + 22 + 3z - 9 = 0$$

$$-x - 11y + 3z = -14$$

OR

$$\boxed{x + 11y - 3z = 14}$$