Northern Cyprus Campus

<table>
<tr><td></td><td colspan="6">Calculus for Functions of Several Variables
Short Exam 1</td><td></td></tr>
<tr><td colspan="3">Code : *Math 120*
Acad.Year: *2011-2012*
Semester : *Fall*
Date : *16.10.2012*
Time : *17:45*
Duration : *45 minutes*</td><td colspan="5">Name: Last Name:
Student No:
Signature:

6 QUESTIONS ON 2 PAGES
TOTAL 42 POINTS</td></tr>
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Show your work! No calculators! Please draw a box around your answers!
Please do not write on your desk!

1. (2 pts.) Find a unit vector in the direction of $\mathbf{a} = (-3, 1, 2)$

$$u = \frac{1}{|a|}\vec{a} = \frac{1}{\sqrt{(-3)^2+1^2+2^2}} \cdot \langle -3,1,2\rangle = \frac{1}{\sqrt{14}}\langle -3,1,2\rangle$$

2. (4 pts.) Give a vector equation for the line through the point $(3, 2, 1)$ that is parallel to the line $\langle -1-4t, 2t-1, 5+t\rangle$.

$v = \langle -4,2,1\rangle \qquad P = (3,2,1)$

$$r(t) = (3,2,1) + t\cdot\langle -4,2,1\rangle = \langle 3-4t, 2t+2, t+1\rangle \quad t\in\mathbb{R}$$

3. (4 pts.) Find an equation of the plane through the point $(2, 3, 0)$ and perpendicular to the vector $\underbrace{\langle -4, 2, -3\rangle}_{n}$.

$$n \cdot \langle x-2, y-3, z-0\rangle = 0 \implies -4(x-2)+3(y-3)-3(z) = 0$$

$$\Pi: \quad -4x+3y-3z = 1$$

4. ($4\times 3 = 12$ pts.) Identify the following surfaces as an *elliptical paraboloid, hyperbolic paraboloid, a hyperboloid of one sheet, a hyperboloid of two sheets, a cone, a circular cylinder, an elliptical cylinder, or a parabolic cylinder*, and identify the axis of symmetry as the x-axis, the y-axis, or the z-axis.

(a) $9x^2+8y^2-3z=0 \quad \equiv 3z = 9x^2+8y^2$

elliptical paraboloid , z-axis

(b) $-6x^2+3y^2-z^2=0 \quad \equiv 3y^2 = 6x^2+z^2$

(elliptical) cone , y-axis

(c) $-5x^2-3y^2+z^2=1$

hyperboloid of 2 sheets , z-axis

(d) $3y^2-9x^2-5z^2+1=0 \quad \equiv 9x^2-3y^2+5z^2=1$

hyperboloid of 1 sheet , y-axis

5. (10 pts.) Determine whether the given lines are parallel, intersecting, or skew. If they intersect, find the intersection point. Show your work.

$$L_1 : x = t + 1,\ y = 2t + 1,\ z = 3t + 2$$

$$L_2 : x = s + 5,\ y = -s + 3,\ z = s + 10$$

$v_1 = (1,2,3)$, $v_2 = (1,-1,1)$

$v_1 \parallel v_2$? $(1,2,3) = k(1,-1,1) \Rightarrow k = 1$ & $k = -2$ $\Rightarrow$ No sol.

So L_1 is not parallel to L_2.

$L_1 \cap L_2 = ?$

$t+1 = s+5$
$2t+1 = -s+3$
$3t+2 = s+10$

Add (1) & (2) $\Rightarrow 3t+2 = 8 \Rightarrow t = 2$

(1) $\Rightarrow 3 = s+5 \Rightarrow \underline{s = -2}$

Try in (2) & (3)

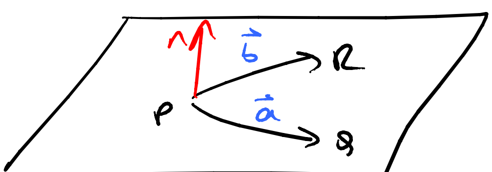
$4+1 = 2+3$ ✓
$8 = -2+10$ ✓

$\Rightarrow (t,s) = (2,-2)$ is a solution to the system

$\Rightarrow$ Put $t=2$ in $L_1 \Rightarrow (3,5,8) \in L_1$
Put $s=-2$ in $L_2 \Rightarrow (3,5,8) \in L_2$

$$\Rightarrow L_1 \cap L_2 = \{(3,5,8)\}$$

6. (10 pts.) Find an equation of the plane that passes through the points $P = (1, 2, 3)$, $Q = (2, 1, -4)$, and $R = (0, 0, 3)$



$\vec{a} = \overrightarrow{PQ} = \langle 1, -1, -7 \rangle$
$\vec{b} = \overrightarrow{PR} = \langle -1, -2, 0 \rangle$

$$\vec{n} = \begin{vmatrix} i & j & k \\ 1 & -1 & -7 \\ -1 & -2 & 0 \end{vmatrix} = \langle -14, +7, -1 \rangle$$

$$\Pi : \langle -14, 7, -1 \rangle \cdot \langle x-0, y-0, z-3 \rangle = 0$$

$$-14x + 7y - z + 3 = 0$$

$$\boxed{14x - 7y + z = 3}$$