M E T U
Northern Cyprus Campus

	Calculus with Analytic Geometry Short Exam 3	
Code : *Math 119* Acad.Year: *2012-2013* Semester : *Summer* Date : *29.07.2013* Time : *17:45* Duration : *30 minutes*	Last Name: Name: KEY Student No: Signature:	
	4 QUESTIONS ON 2 PAGES TOTAL 10 POINTS	

1	2	3	4			

Show your work! No calculators! Please draw a $\boxed{\text{box}}$ around your answers!
Please do not write on your desk!

1. (1 pt.) Determine whether the given statement is true or false. If it is true, explain why. If it is false, explain why or give an example that disproves the statement.

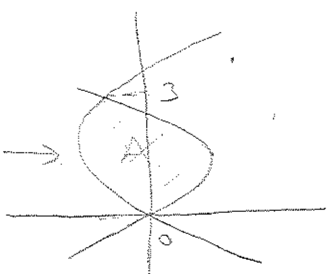
For a function f there exist a number c in $[a, b]$ such that $\int_a^b f(x)\,dx = f(c)(b-a)$.

FALSE. f should be continuous.

e.g. $f(x) = \begin{cases} 0 & x < 0 \\ 1 & x \geq 0 \end{cases}$ $\quad \int_{-1}^{1} f(x)\,dx = 1 \neq f(c)\cdot 2$ for any c in $[-1,1]$.

2. (3 pts.) Find the area of the region bounded by the curves $x = y^2 - 4y$ and $x = 2y - y^2$.

$= y(y-4) \qquad = (2-y)y$



$y^2 - 4y = 2y - y^2$

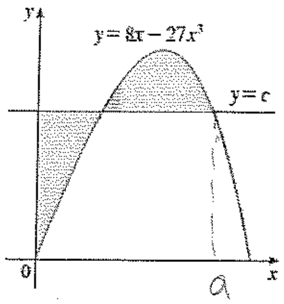
$\Rightarrow 2y^2 - 6y = 0 \Rightarrow 2y(y-3) = 0$

$\Rightarrow y = 0, \; y = 3$

$A = \int_0^3 \left(2y - y^2 - (y^2 - 4y)\right) dy = \int_0^3 (6y - 2y^2)\,dy$

$= 6\frac{y^2}{2} - 2\frac{y^3}{3} \Big|_0^3 = 3\cdot 3^2 - \frac{2}{3}\cdot 3^3 - 0 = \boxed{9}$

3. (3 pts.) The figure shows a horizantal line $y = c$ intersecting the curve $y = 8x - 27x^3$. Find the number c such that the areas of the shaded regions are equal.



$f(a) = c$.

$c =$ average value of f on $[0, a]$.

$$\Rightarrow c = f(a) = \frac{1}{a}\int_0^a f(x)\,dx$$

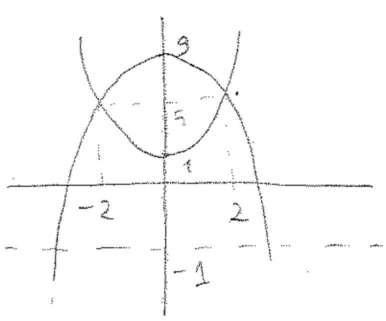
$$\Rightarrow a.(8a - 27a^3) = 4x^2 - 27\frac{x^4}{4}\Big|_0^a$$

$$\Rightarrow 8a^2 - 27a^4 = 4a^2 - \frac{27}{4}a^4 \Rightarrow 4a^2 - 3.\frac{27}{4}a^4 = 0$$

$$a^2(16 - 81a^2) = 0, \quad a > 0 \Rightarrow \sqrt{\frac{16}{81}} = a = \frac{4}{9}$$

$$c = f(a) = 8\cdot\frac{4}{9} - 27\cdot\frac{4^3}{9^3} = \frac{32}{9} - \frac{64}{27} = \boxed{\frac{32}{27}}$$

4. (3 pts.) Write the integral for finding the volume of the solid obtained by rotating the region bounded by the curves $y = x^2 + 1$ and $y = 9 - x^2$ about the line $y = -1$.
DO NOT EVALUATE THIS INTEGRAL.



$$x^2 + 1 = 9 - x^2 \Rightarrow 2x^2 = 8 \Rightarrow x = \mp 2$$

Disk method: $r_{in} = x^2 + 1 + 1$

$r_{out} = 9 - x^2 + 1$

$$V = \int_{-2}^{2} \pi\left((10 - x^2)^2 - (x^2 + 2)^2\right) dx$$

Shell method:

$[1, 5]$

$\sqrt{y-1} - (-\sqrt{y-1})$

$r = y + 1$

$$V = \int_1^5 2\pi(y+1)\,2\sqrt{y-1}\,dy$$

$[5, 9]$

$\sqrt{9-y} - (-\sqrt{9-y})$

$r = y + 1$

$$+ \int_5^9 2\pi(y+1)\,2\sqrt{9-y}\,dy$$