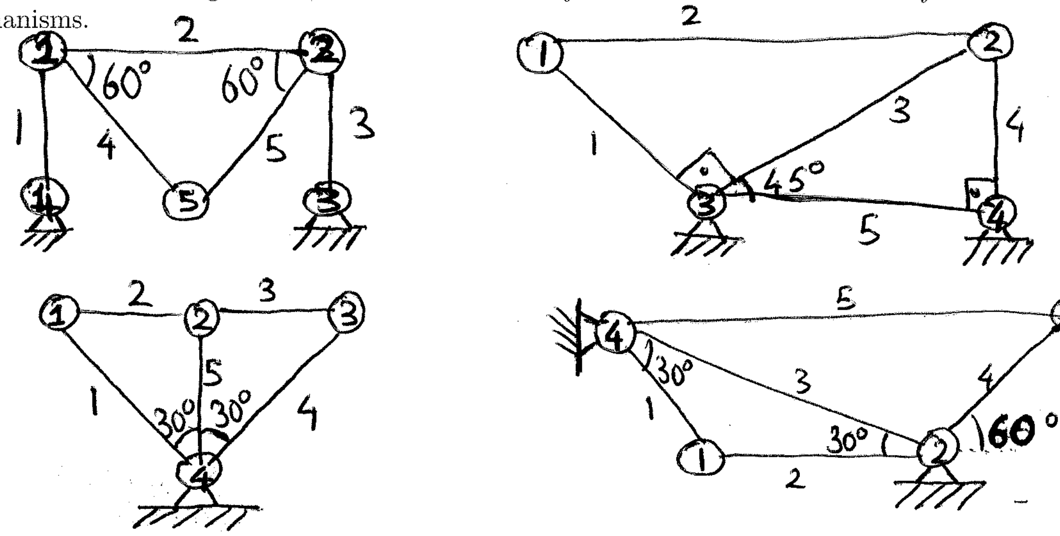
Review Problems II

1.) Given the following trusses, find out whether they are stable or unstable. If they are unstable, find the mechanisms.



2.) Given $< f, g >= \int_0^1 x f(x) g(x) dx$ on continuous functions on $[0, 1]$.

a.) Show that $<,>$ is an inner product.

b.) Find a, b, c such that $\{1, x + a, x^2 + bx + c\}$ is an orthogonal set i.e. each element is orthogonal to the others.

c.) Find the projection of x^3 to the space spanned by $1, x + a, x^2 + bx + c$ found in **(b)**.

3.) **a.)** Find all roots of $z^8 = -1$ and plot them.

b.) Show that if $z_1 z_2$ and $z_1 + z_2$ are both real, then $z_1 = \bar{z}_2$.

c.) Find $\cos(4x)$ in terms of $\cos(x)$ and $\sin(x)$ using complex exponentials.

4.) Compute the Fourier Series of the following functions whose one period is given.

a.) $f(x) = \delta(x + \frac{\pi}{2}) - \delta(x) + \delta(x - \frac{\pi}{2}) \qquad f(x + 2\pi) = f(x)$

b.) $$f(x) = \begin{cases} 0 & \text{for } -\pi \le x \le -\frac{3\pi}{4} \\ 1 & \text{for } -\frac{3\pi}{4} < x < -\frac{\pi}{4} \\ 0 & \text{for } -\frac{\pi}{4} \le x \le \frac{\pi}{4} \\ 1 & \text{for } \frac{\pi}{4} < x < \frac{3\pi}{4} \\ 0 & \text{for } \frac{3\pi}{4} \le x \le \pi \end{cases} \qquad f(x + 2\pi) = f(x)$$

c.) $$f(x) = \begin{cases} 0 & \text{for } -\pi \le x < \pi \\ 2\pi - x & \text{for } \pi \le x < 2\pi \\ -2\pi - x & \text{for } -2\pi \le x < -\pi \end{cases} \qquad f(x + 4\pi) = f(x)$$

d.) $$f(x) = \begin{cases} xe^{-x} & \text{for } 0 < x < \pi \\ 0 & \text{for } \pi \le x \le 2\pi \end{cases} \qquad f(x + 2\pi) = f(x)$$

5.) Say $\int_0^{2\pi} |f(x)|^2 dx = 11$. Suppose that $f(x) = \sum_{k=-\infty}^{\infty} c_k e^{ikx}$, and $c_0 = 1, c_1 = c_{-1} = \frac{1}{2}$.

Show that $|c_k| \leq \sqrt{\frac{1}{2}}$ for all k. (**Hint :** Use Parseval's Identity)

6.) a.) Write F_6 and F_6^{-1}.

b.) Express $\begin{bmatrix} 1 \\ 0 \\ -1 \\ -1 \\ 0 \\ 1 \end{bmatrix}$ as a linear combination of the columns of F_6.

c.) Find the matrices A, B so that

$$F_6 = A \left[\begin{array}{c|c} F_3 & 0 \\ \hline 0 & F_3 \end{array} \right] B$$

7.) Let $f(x) = cos(2x)$

a.) Discretize $f(x)$ for $N = 4$ to get $\vec{f}$

b.) Find the **DFT** of $\vec{f}$ and plot it.

c.) Find the continuous complex FT of $f(x)$ and compare with its **DFT**.

8.) Given $c = (3, 1, -1, 4)$ and $d = (-1, 0, 5, 3)$.

a.) Compute $c * d$ and $c \circledast d$ for $N = 4$

b.) For $N = 4$, write a filter that takes the signal

$\begin{bmatrix} c_0 \\ c_1 \\ c_2 \\ c_3 \end{bmatrix} \longrightarrow \begin{bmatrix} c_0 \\ c_1 \\ 0 \\ 0 \end{bmatrix}$ in the frequency domain.