Review Problems

1.) Consider the given system $A\mathbf{x} = \mathbf{b}$ where $A = \begin{bmatrix} 1 & 2 & 0 \\ 3 & 6 & -1 \\ 1 & 2 & 1 \end{bmatrix}$ and $\mathbf{b} = \begin{bmatrix} 2 \\ 8 \\ 0 \end{bmatrix}$

a.) Find the LU-decomposition of A.

b.) Using the LU-decomposition of A, find the solution to the system $Ax = b$.

2.) Consider the given system $A\mathbf{x} = \mathbf{b}$ where $A = \begin{bmatrix} 2 & 0 & -1 \\ -3 & 0 & 2 \\ -2 & 1 & 0 \end{bmatrix}$ and $\mathbf{b} = \begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix}$

a.) Find A^{-1} if it exists.

b.) Solve the system $A\mathbf{x} = \mathbf{b}$.

3.) Given the differential equation

$$-u''(x) = \delta(x-1) - \delta(x-2) \qquad u'(0) = 0, \quad u'(3) = 0$$

a.) Find the continuous solution. Is it unique? Graph your answer.

b.) Write the difference equations for the same differential equation for n = 2 and solve it. Graph your answer.

3.) Classify the following matrices as **positive-definite, semi-definite, indefinite**.

$$\begin{bmatrix} 1 & 2 \\ 2 & 1 \end{bmatrix} \quad \begin{bmatrix} 1 & 2 & 3 \\ 2 & 2 & 2 \\ 3 & 2 & 1 \end{bmatrix} \quad \begin{bmatrix} 2 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 2 \end{bmatrix} \quad \begin{bmatrix} 1 & -1 & 0 \\ -1 & 0 & 2 \\ 0 & 2 & 3 \end{bmatrix}$$

4.) The matrix $A = \begin{bmatrix} 2 & -2 \\ -2 & 5 \end{bmatrix}$ is positive definite.

a.) Write the energy function of A.

b.) Compute the LDL^T decomposition of A and multiply $\mathbf{x}^T LDL^T \mathbf{x}$ to write the energy function as a sum of squares.

c.) Compute the $Q\Lambda Q^T$ decomposition of A and multiply $\mathbf{x}^T Q\Lambda Q^T \mathbf{x}$ to write the energy function as a sum of squares.

5.) We are given a spring-mass system with three masses $m_1 = m, m_2 = 4m, m_3 = m$ and three identical springs $c_1 = c_2 = c_3 = c$. The upper end is fixed and lower end is free. Assume that the system is in an equilibrium. Find the displacements u_1, u_2, u_3 of the three masses, respectively.

6.) We are given a spring-mass system with two masses $m_1 = 2, m_2 = 6$ and three springs with spring constants $c_1 = 4, c_2 = 6, c_3 = 12$. Both ends are fixed. Assume that there is no friction and external force acting, and the masses are moving up and down.
Find the displacements of the masses $u(t) = \begin{bmatrix} u_1(t) \\ u_2(t) \end{bmatrix}$ at any time t if
$\begin{bmatrix} u_1(0) \\ u_2(0) \end{bmatrix} = \begin{bmatrix} 5 \\ 1 \end{bmatrix}$ and $\begin{bmatrix} u_1'(0) \\ u_2'(0) \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$. Explain the movement of the masses.

7.) An experiment has been conducted to understand the effect of temperature to the output of a certain chemical reaction. The data is given to the right. It is expected that the relation between output and temperature is linear. Find a function $R(T) = A + B \cdot T$ which models the relation between them best..

Experiment Results	
T (Celcius)	R(Gram)
0	3
3	4
5	6
7	10