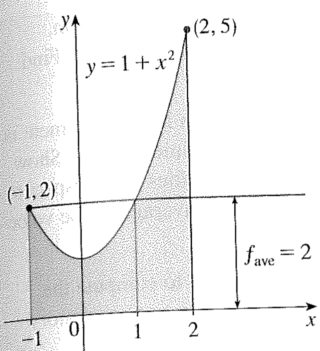


FIGURE 3

In this particular case we can find c explicitly. From Example 1 we know that $f_{ave} = 2$, so the value of c satisfies

$$f(c) = f_{ave} = 2$$

Therefore $\qquad 1 + c^2 = 2 \qquad \text{so} \qquad c^2 = 1$

So in this case there happen to be two numbers $c = \pm 1$ in the interval $[-1, 2]$ that work in the Mean Value Theorem for Integrals.

Examples 1 and 2 are illustrated by Figure 3.

V EXAMPLE 3 Show that the average velocity of a car over a time interval $[t_1, t_2]$ is the same as the average of its velocities during the trip.

SOLUTION If $s(t)$ is the displacement of the car at time t, then, by definition, the average velocity of the car over the interval is

$$\frac{\Delta s}{\Delta t} = \frac{s(t_2) - s(t_1)}{t_2 - t_1}$$

On the other hand, the average value of the velocity function on the interval is

$$v_{ave} = \frac{1}{t_2 - t_1}\int_{t_1}^{t_2} v(t)\,dt = \frac{1}{t_2 - t_1}\int_{t_1}^{t_2} s'(t)\,dt$$

$$= \frac{1}{t_2 - t_1}[s(t_2) - s(t_1)] \qquad \text{(by the Net Change Theorem)}$$

$$= \frac{s(t_2) - s(t_1)}{t_2 - t_1} = \text{average velocity}$$

5.5 Exercises

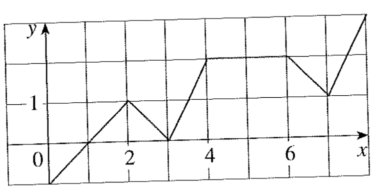
1–8 Find the average value of the function on the given interval.

1. $f(x) = 4x - x^2$, $[0, 4]$
2. $f(x) = \sin 4x$, $[-\pi, \pi]$
3. $g(x) = \sqrt[3]{x}$, $[1, 8]$
4. $g(x) = x^2\sqrt{1 + x^3}$, $[0, 2]$
5. $f(t) = t^2(1 + t^3)^4$, $[0, 2]$
6. $f(\theta) = \sec^2(\theta/2)$, $[0, \pi/2]$
7. $h(x) = \cos^4 x \sin x$, $[0, \pi]$
8. $h(r) = 3/(1 + r)^2$, $[1, 6]$

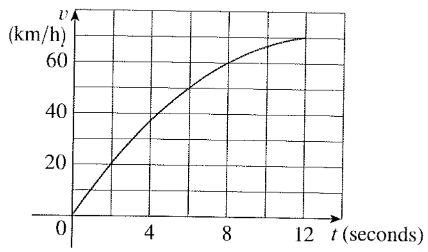
9–12
(a) Find the average value of f on the given interval.
(b) Find c such that $f_{ave} = f(c)$.
(c) Sketch the graph of f and a rectangle whose area is the same as the area under the graph of f.

9. $f(x) = (x - 3)^2$, $[2, 5]$
10. $f(x) = \sqrt{x}$, $[0, 4]$
11. $f(x) = 2 \sin x - \sin 2x$, $[0, \pi]$
12. $f(x) = 2x/(1 + x^2)^2$, $[0, 2]$

13. If f is continuous and $\int_1^3 f(x)\,dx = 8$, show that f takes on the value 4 at least once on the interval $[1, 3]$.
14. Find the numbers b such that the average value of $f(x) = 2 + 6x - 3x^2$ on the interval $[0, b]$ is equal to 3.
15. Find the average value of f on $[0, 8]$.



16. The velocity graph of an accelerating car is shown.



(a) Use the Midpoint rule to estimate the average velocity of the car during the first 12 seconds.
(b) At what time was the instantaneous velocity equal to the average velocity?

17. In a certain city the temperature (in °C) t hours after 9 AM was modeled by the function

$$T(t) = 20 + 6 \sin \frac{\pi t}{12}$$

Find the average temperature during the period from 9 AM to 9 PM.

18. The velocity v of blood that flows in a blood vessel with radius R and length l at a distance r from the central axis is

$$v(r) = \frac{P}{4\eta l}(R^2 - r^2)$$

where P is the pressure difference between the ends of the vessel and η is the viscosity of the blood (see Example 7 in Section 2.7). Find the average velocity (with respect to r) over the interval $0 \leqslant r \leqslant R$. Compare the average velocity with the maximum velocity.

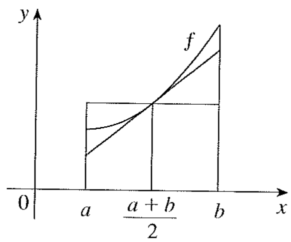
19. The linear density in a rod 8 m long is $12/\sqrt{x+1}$ kg/m, where x is measured in meters from one end of the rod. Find the average density of the rod.

20. If a freely falling body starts from rest, then its displacement is given by $s = \frac{1}{2}gt^2$. Let the velocity after a time T be v_T. Show that if we compute the average of the velocities with respect to t we get $v_{\text{ave}} = \frac{1}{2}v_T$, but if we compute the average of the velocities with respect to s we get $v_{\text{ave}} = \frac{2}{3}v_T$.

21. Use the result of Exercise 57 in Section 4.5 to compute the average volume of inhaled air in the lungs in one respiratory cycle.

22. Use the diagram to show that if f is concave upward on $[a, b]$, then

$$f_{\text{ave}} > f\left(\frac{a+b}{2}\right)$$

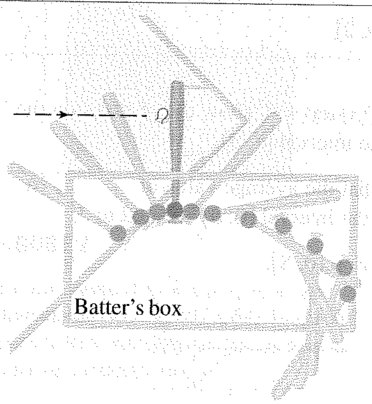


23. Prove the Mean Value Theorem for Integrals by applying the Mean Value Theorem for derivatives (see Section 3.2) to the function $F(x) = \int_a^x f(t)\,dt$.

24. If $f_{\text{ave}}[a, b]$ denotes the average value of f on the interval $[a, b]$ and $a < c < b$, show that

$$f_{\text{ave}}[a, b] = \frac{c-a}{b-a} f_{\text{ave}}[a, c] + \frac{b-c}{b-a} f_{\text{ave}}[c, b]$$

APPLIED PROJECT CALCULUS AND BASEBALL



An overhead view of the position of a baseball bat, shown every fiftieth of a second during a typical swing. (Adapted from *The Physics of Baseball*)

In this project we explore two of the many applications of calculus to baseball. The physical interactions of the game, especially the collision of ball and bat, are quite complex and their models are discussed in detail in a book by Robert Adair, *The Physics of Baseball,* 3d ed. (New York, 2002).

1. It may surprise you to learn that the collision of baseball and bat lasts only about a thousandth of a second. Here we calculate the average force on the bat during this collision by first computing the change in the ball's momentum.

The *momentum* p of an object is the product of its mass m and its velocity v, that is, $p = mv$. Suppose an object, moving along a straight line, is acted on by a force $F = F(t)$ that is a continuous function of time.

(a) Show that the change in momentum over a time interval $[t_0, t_1]$ is equal to the integral of F from t_0 to t_1; that is, show that

$$p(t_1) - p(t_0) = \int_{t_0}^{t_1} F(t)\,dt$$

This integral is called the *impulse* of the force over the time interval.