M E T U
Northern Cyprus Campus

	Calculus with Analytic Geometry Short Exam 1	
Code : *Math 119* Acad.Year: *2013-2014* Semester : *Fall* Date : *28.10.2013* Time : *17:45* Duration : *35 minutes*	Last Name: Name: KEY Signature:	Student No:
	4+1 QUESTIONS 2 PAGES TOTAL 20 + 2 POINTS	

1(4)	2(6)	3(4)	4(6)	B(2)			

Show your work! No calculators! Please draw a $\boxed{\text{box}}$ around your answers!
Please do not write on your desk!

1. (2 × 2 = 4 pts.) Evaluate the limit, if it exists. **Give reasoning.**

(a) $\displaystyle\lim_{x\to 1}\frac{x-1}{x^2+2x-3} = \lim_{\substack{x\to 1\\(x\neq 1)}}\frac{x-1}{(x-1)(x+3)} = \lim_{x\to 1}\frac{1}{x+3} = \frac{1}{4}$

(b) $\displaystyle\lim_{x\to 4^-}\frac{1}{x-4}-\frac{1}{|x-4|} = \lim_{\substack{x\to 4^-\\(x<4)}}\frac{1}{x-4}-\frac{1}{-(x-4)} = \lim_{x\to 4^-}\frac{2}{x-4} = -\infty$

$|x-4| = \begin{cases} x-4 & x\geq 4 \\ -(x-4) & x<4 \end{cases}$

($x-4$: $-$ | 4 | $+$)

2. (6 pts.) Find where $f(x)$ is continuous. **Give reasoning.**

$$f(x) = \begin{cases} \frac{1}{4-x^2} & \text{if } x<0 \\ \frac{1-x}{4} & \text{if } 0\leq x<1 \\ 2x-2 & \text{if } x>1 \end{cases}$$

$\frac{1}{4-x^2}$ is cont everywhere except for $x=\mp 2$. So f is cont on $(-\infty,-2)\cup(-2,0)$. (Note that if $x>1$, $f(x)=2x-2$)

$\frac{1-x}{4}$ and $2x-2$ are polyn, so cont everywhere. So f is cont on $(0,1)\cup(1,\infty)$.

Check $x=0$ and $x=1$.

$\displaystyle\lim_{x\to 0^+}f(x) = \lim_{x\to 0^+}\frac{1-x}{4} = \frac{1}{4} = \lim_{x\to 0^-}f(x) = \lim_{x\to 0^-}\frac{1}{4-x^2} = \frac{1}{4} = f(0)$

So f is cont at $x=0$.

f is not defined at 1, so f is not cont at $x=1$.

So f is cont on $(-\infty,-2)\cup(-2,1)\cup(1,\infty)$.

3. (4 pts.) Consider $3x^2 + 2xy + 3y^2 = 1$

(a) Find $\left.\frac{dy}{dx}\right|_{(0,\frac{1}{\sqrt{3}})}$ using implicit differentiation by regarding y as a function of x.

$6x + 2y + 2x\frac{dy}{dx} + 6y\frac{dy}{dx} = 0$. $(x,y) = (0, \frac{1}{\sqrt{3}}) \Rightarrow 0 + \frac{2}{\sqrt{3}} + 0 + \frac{6}{\sqrt{3}} \cdot \left.\frac{dy}{dx}\right|_{(0,\frac{1}{\sqrt{3}})} = 0$

$\Rightarrow -\frac{2}{\sqrt{3}} = \frac{6}{\sqrt{3}} \left.\frac{dy}{dx}\right|_{(0,\frac{1}{\sqrt{3}})} \Rightarrow \boxed{\left.\frac{dy}{dx}\right|_{(0,\frac{1}{\sqrt{3}})} = \frac{-1}{3}}$

(b) Find $\left.\frac{dx}{dy}\right|_{(-\frac{1}{\sqrt{3}},0)}$ using implicit differentiation by regarding ~~y~~ x as a function of ~~x~~ y

$6x\frac{dx}{dy} + 2\frac{dx}{dy}y + 2x + 6y = 0$

$(x,y) = (\frac{-1}{\sqrt{3}}, 0) \Rightarrow \frac{-6}{\sqrt{3}} \left.\frac{dx}{dy}\right|_{(\frac{-1}{\sqrt{3}},0)} + 0 - \frac{2}{\sqrt{3}} + 0 = 0 \Rightarrow \boxed{\left.\frac{dx}{dy}\right|_{(\frac{-1}{\sqrt{3}},0)} = \frac{-1}{3}}$

4. (6 pts.) Using the definition of the limit, prove that $\lim_{x\to 3} 11 - 5x = -4$.

Given $\varepsilon > 0$, we need to find $\delta > 0$ st

$0 < |x-3| < \delta \Rightarrow |f(x) - (-4)| < \varepsilon$.

$|f(x) + 4| = |11 - 5x + 4| = |15 - 5x| = 5|3 - x| = 5|x-3| < 5\delta$

So choose $\delta = \frac{\varepsilon}{5}$.

For any $\varepsilon > 0$, let $\delta = \frac{\varepsilon}{5}$. If $0 < |x-3| < \delta$ then

$|f(x) + 4| = 5|x-3| < 5\delta = 5\frac{\varepsilon}{5} = \varepsilon$.

5. Bonus ($1 + 1 = 2$ pts.) Determine whether the given statement is true or false. No explanations required.

(a) The definition of $\lim_{x\to c} f(x) = L$ is

For all $\epsilon > 0$, there is a $\delta > 0$ such that whenever $|f(x) - L| < \epsilon$, we must have $0 < |x - c| < \delta$.

FALSE

(b) The primary online communication form in this course is the announcements tab on http://www.math.ncc.metu.edu.tr/content/courses/math119/ .

TRUE