M E T U
Northern Cyprus Campus

Calculus With Analytic Geometry
Short Exam 1

Code : *Math 119*	Last Name:
Acad.Year: *2011-2012*	Name:
Semester : *Spring*	Department:
Date : *16.4.2012*	Student No:
Time : *18:45*	Section:
Duration : *45 minutes*	Signature:
	Recitation:

4 QUESTIONS ON 4 PAGES
TOTAL 50 POINTS

1	2	3	4	

Show your work! No calculators! Please draw a box around your answers!
Please do not write on your desk!

1. (4 + 6 = 10 pts.) We want to prove that the equation $x^5 + x^3 + 2x = \cos(x)$ has **only one** real solution.

   (a) Prove that this equation has **at least one** solution.

Let $f(x) = x^5 + x^3 + 2x - \cos x$. Then we are trying to find $f(x) = 0$

$f(0) = 0+0+0-1 = -1 < 0$ & $f(1) = 1 + 1 + \underbrace{2 - \cos(1)}_{\geq 0} > 0$

Since f is continuous, by IVT there is a $c \in (0,1)$ with $f(c) = 0$

Hence the equation has at least one solution.

   (b) Prove that this equation has **at most one** solution.

Suppose there are two solutions, $a < b$. Then

$\frac{f(b)-f(a)}{b-a} = \frac{0-0}{b-a} = 0 = f'(d)$ for some $d \in (a,b)$ by MVT.

But $f'(x) = \underbrace{5x^4}_{\geq 0} + \underbrace{3x^2}_{\geq 0} + \underbrace{2 - \sin x}_{> 0} > 0$. Hence $f'(d) = 0 > 0$, contradiction.

Therefore, f cannot have more than one zero.

That means the original equation has at most one solution.

2. (8 pts.) Find two numbers A and B (with $A \leq B$) whose difference is 10 and whose product AB is minimized.

$B - A = 10$, $P = AB$ ← minimize $\Rightarrow P(A) = A(10+A) = A^2 + 10A$

$P'(A) = 2A + 10 = 0 \iff A = -5$ ← the only critical point.

		-5	
$P'(A)$	$-$	0	$+$
P	dec	local min	inc

$\Rightarrow$ Absolute min because this is the only critical point.

$\Rightarrow A = -5$, $B = 5$ Product is minimized with value -25.

3. (24 pts.) Let $f(x) = \dfrac{x}{(x+1)^2}$.

(a) (2 pts.) Find the domain of $f(x)$.

$$\text{Domain}(f) = \mathbb{R} \setminus \{-1\} = (-\infty, -1) \cup (-1, \infty)$$

(b) (2 pts.) Find the x–intercept(s) and y–intercept(s) of $f(x)$.

$$f(0) = 0 \Rightarrow y\text{-intercept} = (0,0)$$

$$f(x) = 0 \Leftrightarrow x = 0 \Rightarrow x\text{-intercept} = (0,0)$$

(c) (2 pts.) Find the horizontal and vertical asymptote(s) of $f(x)$ if there are any.

$$\lim_{x \to \pm\infty} \frac{x}{(x+1)^2} = 0 \longrightarrow y = 0 \text{ is the only horizontal asymptote}$$

$$\lim_{x \to -1^{\pm}} \frac{x}{(x+1)^2} = -\infty \longrightarrow x = -1 \text{ is the only vertical asymptote}$$

(d) (4 pts.) Find the derivative of $f(x)$, $f'(x)$.

$$f'(x) = \frac{1 \cdot (x+1)^2 - x \cdot 2 \cdot (x+1) \cdot 1}{((x+1)^2)^2} = \frac{x+1-2x}{(x+1)^3}$$

$$f'(x) = \frac{1-x}{(x+1)^3}$$

(e) (4 pts.) Find the interval(s) of increase and decrease for $f(x)$.

$$f'(x) = \frac{1-x}{(x+1)^3} = 0 \Leftrightarrow 1 - x = 0 \Leftrightarrow x = 1$$

		-1		1	
$f'(x)$	$-$		$+$	0	$-$
$f(x)$	dec		inc		dec
				local max.	

f is increasing on $(-1, 1)$

f is decreasing on $(-\infty, -1), (1, \infty)$

(f) (4 pts.) Find the second derivative of $f(x)$, $f''(x)$.

$$f''(x) = \frac{(-1)\cdot(x+1)^3 - (1-x)\cdot 3\cdot(x+1)^2\cdot 1}{((x+1)^3)^2} = \frac{-x-1-3+3x}{(x+1)^4}$$

$$f''(x) = \frac{2x-4}{(x+1)^4}$$

(g) (4 pts.) Find the interval(s) of concavity for $f(x)$.

$$f''(x) = \frac{2x-4}{(x+1)^4} = 0 \iff x = 2$$

	-1		2	
$f''(x)$	$-$	$-$	0	$+$
f	C.D.	C.D.	inf	C.U.

f is concave up on $(2, \infty)$

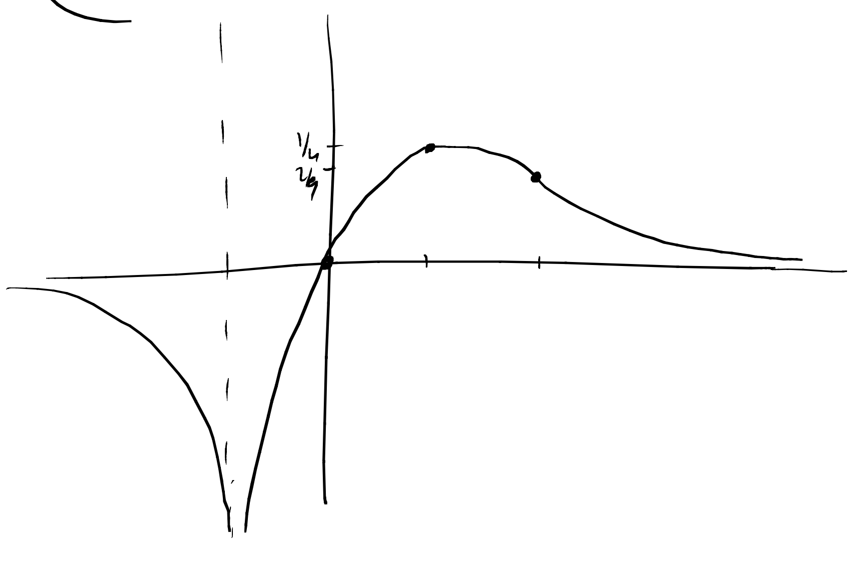
f is concave down on $(-\infty, -1), (-1, 2)$

(h) (2 pts.) By using the above information, draw the graph of $f(x)$.

	-1		1	2
f	dec	inc	dec	dec
	CD	CD	CD	CU

$f(1) = \frac{1}{4}$

$f(2) = \frac{2}{9}$



4. (8 pts.) Find **the** function $f(t)$ that satisfies following conditions.

$$f'(t) = 2t - \underbrace{3\sqrt{t}}_{3t^{1/2}} + \underbrace{\frac{4}{\sqrt[3]{t}}}_{4t^{-1/3}} - \cos(t) + 1 \qquad ; \qquad f(1) = 6$$

General form of $f(t)$ is

$$f(t) = 2\frac{t^2}{2} - 3\frac{t^{3/2}}{3/2} + 4\frac{t^{2/3}}{2/3} - \sin(t) + t + C$$

$$f(t) = t^2 - 2t^{3/2} + 6t^{2/3} - \sin(t) + t + C$$

$$f(1) = 1 - 2 + 6 - \sin(1) + 1 + C = 6$$

$$6 - \sin(1) + C = 6$$

$$C = \sin(1)$$

$$f(t) = t^2 - 2t^{3/2} + 6t^{2/3} - \sin(t) + t + \sin(1)$$

is the function that satisfies both equations.