M E T U
Northern Cyprus Campus

	Calculus With Analytic Geometry Short Exam 2	

Code : *Math 119* Acad.Year : *2012-2013* Semester : *Fall* Date : *12.12.2012* Time : *18:45* Duration : *45 minutes*	Last Name: Department: Section:	Name: Student No: Signature:
	4 QUESTIONS ON 2 PAGES TOTAL 42+2 POINTS	

1	2	3	4			

Show your work! No calculators! Please draw a $\boxed{\text{box}}$ around your answers!
Please do not write on your desk!

1. ($3 \times 5 = 15$ pts.) Find the following integrals.

(a) $\int_0^1 x^2\sqrt{9x+10}\,dx$ $= \int_{10}^{19} \left(\frac{u-10}{9}\right)^2 \sqrt{u}\,\frac{du}{9} = \frac{1}{9^3}\int_{10}^{19}(u^2\sqrt{u} - 20u\sqrt{u} + 100\sqrt{u})du$

$9x+10=u \Rightarrow 9dx = du$

$x=\frac{u-10}{9} \Rightarrow x^2 = \left(\frac{u-10}{9}\right)^2$

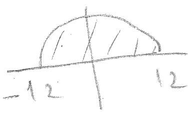
$x=0 \Rightarrow u=10,\ x=1 \Rightarrow u=19$

$= \frac{1}{9^3}\left(\frac{2}{7}u^{7/2} - 20.\frac{2}{5}u^{5/2} + 100\frac{2}{3}u^{3/2}\Big|_{10}^{19}\right)$

$= \frac{1}{9^3}\left(\left(\frac{2}{7}(19)^{7/2} - 8(19)^{5/2} + \frac{200}{3}(19)^{3/2}\right) - \left(\frac{2}{7}.10^{7/2} - 8.10^{5/2} + \frac{200}{3}(10)^{3/2}\right)\right)$

(b) $\int_{-12}^{12} -3\sqrt{144-x^2}\,dx$

$y=\sqrt{144-x^2}$ is the upper semicircle with center $(0,0)$ radius 12.



$\int_{-12}^{12} -3\sqrt{144-x^2}\,dx = -3\int_{-12}^{12}\sqrt{144-x^2}\,dx = -3.\frac{\pi(12)^2}{2} = \boxed{-216\pi}$

(c) $\int \frac{\cos(z)}{\sin^\pi(z)}\,dz$ $= \int \frac{du}{u^\pi} = \frac{u^{-\pi+1}}{-\pi+1} + c = \boxed{\frac{(\sin z)^{1-\pi}}{1-\pi} + c}$

$\sin z = u \Rightarrow \cos z\,dz = du$

2. (8 pts.) Find $\frac{dh}{dx}$ if $h(x) = \int_{\sqrt{x}}^{x^6} \sqrt{t}\sin t^4 dt.$

$h(x) = \int_{\sqrt{x}}^{x_0} \sqrt{t}\sin t^4 dt + \int_{x_0}^{x^6} \sqrt{t}\sin t^4 dt = -\int_{x_0}^{\sqrt{x}} \sqrt{t}\sin t^4 dt + \int_{x_0}^{x^6} \sqrt{t}\sin t^4 dt$

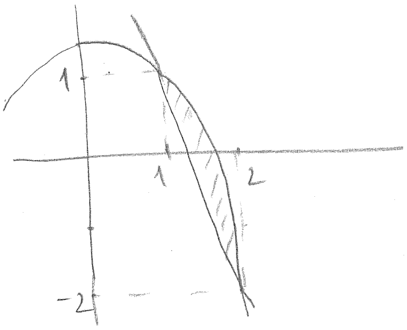
$\Rightarrow \frac{dh}{dx}(x) = \left(-\sqrt[4]{x}\sin x^2\right)\frac{1}{2\sqrt{x}} + \left(x^3\sin x^{24}\right)6x^5$

3. (5 + 7 + 7 = 19 pts.) Let R be the region bounded by the curves $f(x) = 4 - 3x$ and $g(x) = 2 - x^2$. Let S be the solid obtained by rotating the region R about the line $x = 3$.

(a) Find *the area of R*.

$4-3x = 2-x^2 \iff x^2-3x+2=0 \iff (x-1)(x-2)=0 \iff x=1$ or $x=2$.

$(f(1)=1,\ f(2)=-2)$

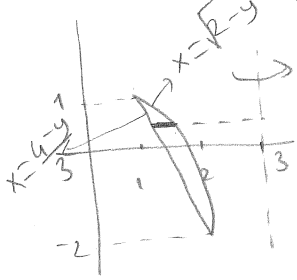


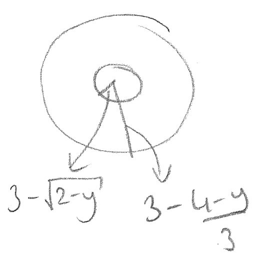
$$\text{Area of } R = \int_1^2 \big((2-x^2)-(4-3x)\big)\,dx$$

$$= \int_1^2 (-2-x^2+3x)\,dx = -2x - \frac{x^3}{3} + \frac{3x^2}{2}\Big|_1^2$$

$$= -4 - \frac{8}{3} + 6 - \left(-2-\frac{1}{3}+\frac{3}{2}\right) = \frac{1}{6}$$

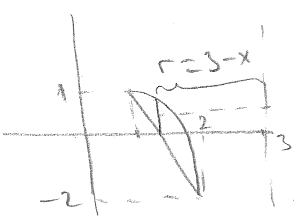
(b) **Write the integral** to obtain *the volume of S* **using the method of disks or washers.** DO NOT EVALUATE THIS INTEGRAL.





$$\int_{-2}^{1} \pi\left(\left(3-\frac{4-y}{3}\right)^2 - \left(3-\sqrt{2-y}\right)^2\right) dy$$

(c) **Write the integral** to obtain *the volume of S* **using the method of shells.** DO NOT EVALUATE THIS INTEGRAL.



$\} \ 2-x^2-(4-3x)$

$r = 3-x$

$$\int_1^2 2\pi(3-x)(2-x^2-4+3x)\,dx$$

$$= \int_1^2 2\pi(3-x)(-x^2+3x-2)\,dx$$

4. (2 pts.) What is the e-mail address of your teaching assistant?

abozer@metu.edu.tr or mucelik@metu.edu.tr