M E T U
Northern Cyprus Campus

	Calculus for Functions of Several Variables **Short Exam 3**	

Code : *Math 120* Acad.Year: *2011-2012* Semester : *Spring* Date : *22.5.2012* Time : *17:45* Duration : *45 minutes*	Last Name: Name: Department: Student No: Section: Signature: Recitation: 3 QUESTIONS ON 3 PAGES TOTAL 50 POINTS

1	2	3	4				

Show your work! No calculators! Please draw a $\boxed{\text{box}}$ around your answers!

Please do not write on your desk!

1. (6 pts.) Evaluate the following limits.

(a) $\lim\limits_{n\to\infty} (e^{2n} + 6n)^{1/n}$

Consider $\lim\limits_{x\to\infty} \underbrace{(e^{2x}+6x)^{1/x}}_{y}$

$$\lim_{x\to\infty} \ln y = \lim_{x\to\infty} \frac{\ln(e^{2x}+6x)}{x} \overset{\infty/\infty}{=} \lim_{x\to\infty} \frac{\frac{1}{e^{2x}+6x}\cdot(2e^{2x}+6)}{1}$$

$$= \lim_{x\to\infty} \frac{2e^{2x}+6}{e^{2x}+6x} = \lim_{x\to\infty} \frac{2+6\cdot e^{-2x}}{1+6xe^{-2x}} = \frac{2+0}{1+0} = 2$$

$$\Rightarrow \lim_{x\to\infty} y = \lim_{x\to\infty} e^{\ln y} = e^{\lim\limits_{x\to\infty} \ln y} = e^2 \Rightarrow \lim_{n\to\infty} (e^{2n}+6n)^{1/n} = e^2$$

(b) $\lim\limits_{n\to\infty} \frac{1}{n}$ $= 0$

$$\left(\text{Important Note : } \frac{1}{n} \neq \sum_{n=1}^{\infty} \frac{1}{n} \right)$$

2. ($8 \times 4 = 32$ pts.) Fill in the blanks according to the rules specified below.
In the first blank space, write "C" for convergent or "D" for Divergent.
In the second blank space, provide the name of the series test you have used.
If your answer for the test name is "Integral Test", "Comparison Test", or "Limit Comparison Test", then, in the third blank space provide the integral or series you have compared.

Example :

Series	Convergent or Divergent	Name of the test	Compared with...
$\sum\limits_{n=1}^{\infty} \frac{1}{n}$	Divergent	$p-$test	—

Series	Convergent or Divergent	Name of the test	Compared with...
$\displaystyle\sum_{n=1}^{\infty}\frac{\cos(n\pi)}{n}$ $(-1)^n$	CONV.	Alternating Series Test	—
$\displaystyle\sum_{n=1}^{\infty}\frac{22n+5}{(-1)^n}$	DIV.	Test for Divergence	
$\displaystyle\sum_{n=1}^{\infty}\frac{\sin(\frac{n\pi}{2})}{n^2} = \sum_{n=1}^{\infty}\frac{(-1)^n}{(2n-1)^2}$	CONV.	Alternating Series Test	—
$\displaystyle\sum_{n=1}^{\infty}\frac{n}{n^3+4}$	CONV.	Comparison	$\frac{n}{n^3+4} < \frac{1}{n^2}$
$\displaystyle\sum_{n=2}^{\infty}\frac{1}{n\ln(n)}$	DIV.	Integral Test	$\int_2^{\infty}\frac{1}{x\ln x}\,dx$
$\displaystyle\sum_{n=1}^{\infty}\frac{\sqrt[3]{n^7-3}}{\sqrt[4]{n^{17}+2012}}$	CONV.	Limit Comparison	$\frac{1}{n^{17/4-7/3}}$
$\displaystyle\sum_{n=2}^{\infty}\frac{1}{\ln(n)}$	DIV.	Comparison	$\frac{1}{\ln n} > \frac{1}{n}$
$\displaystyle\sum_{n=1}^{\infty}\left(\frac{5}{e\pi}\right)^{2n-1}$	CONV.	GEO. SER	$\left\vert\frac{5}{e\pi}\right\vert < 1$

3. ($3 \times 4 = 12$ pts.) Determine if the given series is **absolutely convergent**, **conditionally convergent**, or **divergent**. Give reasoning.

(a) $\displaystyle\sum_{n=-3}^{\infty} \frac{\cos(\frac{n\pi}{2})}{n+4} = 0 - \frac{1}{2} + 0 + \frac{1}{4} + 0 - \frac{1}{6} + \cdots = \sum_{n=1}^{\infty} (-1)^n \frac{1}{2n}$

$\displaystyle\sum_{n=1}^{\infty} \frac{1}{2n}$ is divergent by p-test.

$\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^n}{2n}$ is convergent by A.S.T. :

Alternating ✓

$\frac{1}{2n}$ decreasing ✓

$\lim \frac{1}{2n} = 0$ ✓

The given series is $\boxed{\text{conditionally convergent.}}$

(b) $\displaystyle\sum_{n=1}^{\infty} \frac{\sin(2n)}{(2n+1)^2}$

$\left|\frac{\sin(2n)}{(2n+1)^2}\right| \leq \frac{1}{(2n+1)^2} \leq \frac{1}{4n^2}$

$\displaystyle\sum_{n=1}^{\infty} \frac{1}{4n^2}$ is convergent by p-test

$\Rightarrow \displaystyle\sum_{n=1}^{\infty} \left|\frac{\sin 2n}{(2n+1)^2}\right|$ is convergent by comparison test.

$\Rightarrow \displaystyle\sum_{n=1}^{\infty} \frac{\sin 2n}{(2n+1)}$ is $\boxed{\text{absolutely convergent}}$

(c) $\displaystyle\sum_{n=2}^{\infty} \frac{(-1)^n}{n\ln(n)}$

$\displaystyle\sum_{n=2}^{\infty} \frac{1}{n\ln n}$ is DIV. by integral test

$\displaystyle\sum_{n=2}^{\infty} \frac{(-1)^n}{n\ln n}$ is convergent by Alternating Series Test,

- Alternating ✓
- $\frac{1}{n\ln n}$ decreasing since $n\ln n$ is inc. ✓
- $\lim \frac{1}{n\ln n} = 0$ ✓

$\displaystyle\sum_{n=2}^{\infty} \frac{(-1)^n}{n\ln n}$ is $\boxed{\text{conditionally convergent.}}$