EXAMPLE 7 Figure 4 shows the power consumption in the city of San Francisco for a day in September (P is measured in megawatts; t is measured in hours starting at midnight). Estimate the energy used on that day.

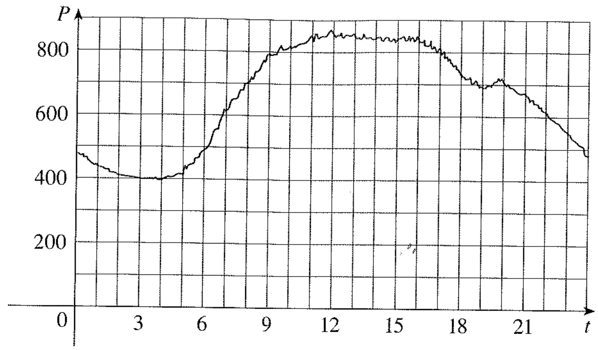


FIGURE 4

SOLUTION Power is the rate of change of energy: $P(t) = E'(t)$. So, by the Net Change Theorem,

$$\int_0^{24} P(t)\,dt = \int_0^{24} E'(t)\,dt = E(24) - E(0)$$

is the total amount of energy used on that day. We approximate the value of the integral using the Midpoint Rule with 12 subintervals and $\Delta t = 2$:

$$\begin{aligned}\int_0^{24} P(t)\,dt &\approx [P(1) + P(3) + P(5) + \cdots + P(21) + P(23)]\,\Delta t\\ &\approx (440 + 400 + 420 + 620 + 790 + 840 + 850\\ &\qquad + 840 + 810 + 690 + 670 + 550)(2)\\ &= 15{,}840\end{aligned}$$

The energy used was approximately 15,840 megawatt-hours. ■

A note on units

How did we know what units to use for energy in Example 7? The integral $\int_0^{24} P(t)\,dt$ is defined as the limit of sums of terms of the form $P(t_i^*)\,\Delta t$. Now $P(t_i^*)$ is measured in megawatts and Δt is measured in hours, so their product is measured in megawatt-hours. The same is true of the limit. In general, the unit of measurement for $\int_a^b f(x)\,dx$ is the product of the unit for $f(x)$ and the unit for x.

4.4 Exercises

1–4 Verify by differentiation that the formula is correct.

1. $\displaystyle\int \frac{x}{\sqrt{x^2+1}}\,dx = \sqrt{x^2+1} + C$

2. $\displaystyle\int \cos^2 x\,dx = \tfrac{1}{2}x + \tfrac{1}{4}\sin 2x + C$

3. $\displaystyle\int \cos^3 x\,dx = \sin x - \tfrac{1}{3}\sin^3 x + C$

4. $\displaystyle\int \frac{x}{\sqrt{a+bx}}\,dx = \frac{2}{3b^2}(bx - 2a)\sqrt{a+bx} + C$

5–16 Find the general indefinite integral.

5. $\displaystyle\int (x^2 + x^{-2})\,dx$

6. $\displaystyle\int (\sqrt{x^3} + \sqrt[3]{x^2})\,dx$

7. $\int \left(x^4 - \frac{1}{2}x^3 + \frac{1}{4}x - 2\right) dx$

8. $\int (y^3 + 1.8y^2 - 2.4y)\, dy$

9. $\int (u + 4)(2u + 1)\, du$

10. $\int v(v^2 + 2)^2\, dv$

11. $\int \frac{x^3 - 2\sqrt{x}}{x}\, dx$

12. $\int \left(u^2 + 1 + \frac{1}{u^2}\right) du$

13. $\int (\theta - \csc\theta \cot\theta)\, d\theta$

14. $\int \sec t\,(\sec t + \tan t)\, dt$

15. $\int (1 + \tan^2\alpha)\, d\alpha$

16. $\int \frac{\sin 2x}{\sin x}\, dx$

17–18 Find the general indefinite integral. Illustrate by graphing several members of the family on the same screen.

17. $\int \left(\cos x + \frac{1}{2}x\right) dx$

18. $\int (1 - x^2)^2\, dx$

19–42 Evaluate the integral.

19. $\int_0^2 (6x^2 - 4x + 5)\, dx$

20. $\int_1^3 (1 + 2x - 4x^3)\, dx$

21. $\int_{-2}^0 \left(\frac{1}{2}t^4 + \frac{1}{4}t^3 - t\right) dt$

22. $\int_0^3 (1 + 6w^2 - 10w^4)\, dw$

23. $\int_0^2 (2x - 3)(4x^2 + 1)\, dx$

24. $\int_{-1}^1 t(1 - t)^2\, dt$

25. $\int_0^\pi (4\sin\theta - 3\cos\theta)\, d\theta$

26. $\int_1^2 \left(\frac{1}{x^2} - \frac{4}{x^3}\right) dx$

27. $\int_1^4 \left(\frac{4 + 6u}{\sqrt{u}}\right) du$

28. $\int_1^2 \left(x + \frac{1}{x}\right)^2 dx$

29. $\int_1^4 \sqrt{\frac{5}{x}}\, dx$

30. $\int_1^9 \frac{3x - 2}{\sqrt{x}}\, dx$

31. $\int_1^4 \sqrt{t}\,(1 + t)\, dt$

32. $\int_{\pi/4}^{\pi/3} \csc^2\theta\, d\theta$

33. $\int_0^{\pi/4} \frac{1 + \cos^2\theta}{\cos^2\theta}\, d\theta$

34. $\int_0^{\pi/3} \frac{\sin\theta + \sin\theta\tan^2\theta}{\sec^2\theta}\, d\theta$

35. $\int_1^{64} \frac{1 + \sqrt[3]{x}}{\sqrt{x}}\, dx$

36. $\int_1^8 \frac{x - 1}{\sqrt[3]{x^2}}\, dx$

37. $\int_0^1 \left(\sqrt[4]{x^5} + \sqrt[5]{x^4}\right) dx$

38. $\int_0^1 (1 + x^2)^3\, dx$

39. $\int_2^5 |x - 3|\, dx$

40. $\int_0^2 |2x - 1|\, dx$

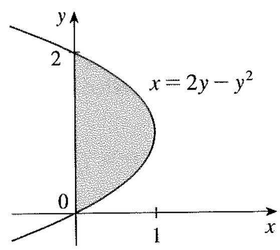
41. $\int_{-1}^2 (x - 2|x|)\, dx$

42. $\int_0^{3\pi/2} |\sin x|\, dx$

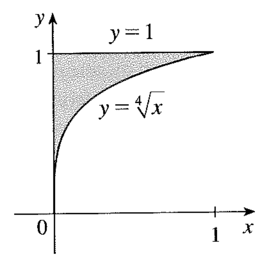
43. Use a graph to estimate the x-intercepts of the curve $y = 1 - 2x - 5x^4$. Then use this information to estimate the area of the region that lies under the curve and above the x-axis.

44. Repeat Exercise 43 for the curve $y = 2x + 3x^4 - 2x^6$.

45. The area of the region that lies to the right of the y-axis and to the left of the parabola $x = 2y - y^2$ (the shaded region in the figure) is given by the integral $\int_0^2 (2y - y^2)\, dy$. (Turn your head clockwise and think of the region as lying below the curve $x = 2y - y^2$ from $y = 0$ to $y = 2$.) Find the area of the region.



46. The boundaries of the shaded region are the y-axis, the line $y = 1$, and the curve $y = \sqrt[4]{x}$. Find the area of this region by writing x as a function of y and integrating with respect to y (as in Exercise 45).



47. If $w'(t)$ is the rate of growth of a child in kilograms per year, what does $\int_5^{10} w'(t)\, dt$ represent?

48. The current in a wire is defined as the derivative of the charge: $I(t) = Q'(t)$. (See Example 3 in Section 2.7.) What does $\int_a^b I(t)\, dt$ represent?

49. If oil leaks from a tank at a rate of $r(t)$ liters per minute at time t, what does $\int_0^{120} r(t)\, dt$ represent?

50. A honeybee population starts with 100 bees and increases at a rate of $n'(t)$ bees per week. What does $100 + \int_0^{15} n'(t)\, dt$ represent?

51. In Section 3.7 we defined the marginal revenue function $R'(x)$ as the derivative of the revenue function $R(x)$, where x is the number of units sold. What does $\int_{1000}^{5000} R'(x)\, dx$ represent?

52. If $f(x)$ is the slope of a trail at a distance of x kilometers from the start of the trail, what does $\int_3^5 f(x)\, dx$ represent?

53. If x is measured in meters and $f(x)$ is measured in newtons, what are the units for $\int_0^{100} f(x)\, dx$?

54. If the units for x are feet and the units for $a(x)$ are pounds per foot, what are the units for da/dx? What units does $\int_2^8 a(x)\,dx$ have?

55–56 The velocity function (in meters per second) is given for a particle moving along a line. Find (a) the displacement and (b) the distance traveled by the particle during the given time interval.

55. $v(t) = 3t - 5, \quad 0 \leqslant t \leqslant 3$

56. $v(t) = t^2 - 2t - 8, \quad 1 \leqslant t \leqslant 6$

57–58 The acceleration function (in m/s^2) and the initial velocity are given for a particle moving along a line. Find (a) the velocity at time t and (b) the distance traveled during the given time interval.

57. $a(t) = t + 4, \quad v(0) = 5, \quad 0 \leqslant t \leqslant 10$

58. $a(t) = 2t + 3, \quad v(0) = -4, \quad 0 \leqslant t \leqslant 3$

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59. The linear density of a rod of length 4 m is given by $\rho(x) = 9 + 2\sqrt{x}$ measured in kilograms per meter, where x is measured in meters from one end of the rod. Find the total mass of the rod.

60. Water flows from the bottom of a storage tank at a rate of $r(t) = 200 - 4t$ liters per minute, where $0 \leqslant t \leqslant 50$. Find the amount of water that flows from the tank during the first 10 minutes.

61. The velocity of a car was read from its speedometer at 10-second intervals and recorded in the table. Use the Midpoint Rule to estimate the distance traveled by the car.

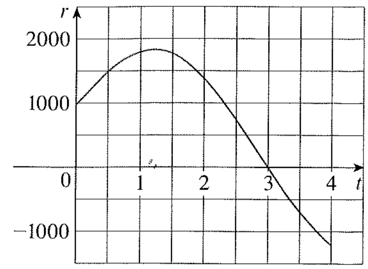
t (s)	v (mi/h)	t (s)	v (mi/h)
0	0	60	56
10	38	70	53
20	52	80	50
30	58	90	47
40	55	100	45
50	51		

62. Suppose that a volcano is erupting and readings of the rate $r(t)$ at which solid materials are spewed into the atmosphere are given in the table. The time t is measured in seconds and the units for $r(t)$ are tonnes (metric tons) per second.

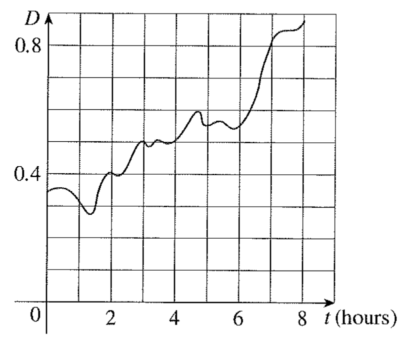
t	0	1	2	3	4	5	6
$r(t)$	2	10	24	36	46	54	60

(a) Give upper and lower estimates for the total quantity $Q(6)$ of erupted materials after 6 seconds.
(b) Use the Midpoint Rule to estimate $Q(6)$.

63. Water flows into and out of a storage tank. A graph of the rate of change $r(t)$ of the volume of water in the tank, in liters per day, is shown. If the amount of water in the tank at time $t = 0$ is 25,000 L, use the Midpoint Rule to estimate the amount of water in the tank four days later.



64. Shown is the graph of traffic on an Internet service provider's T1 data line from midnight to 8:00 AM. D is the data throughput, measured in megabits per second. Use the Midpoint Rule to estimate the total amount of data transmitted during that time period.



65. Shown is the power consumption in the province of Ontario, Canada, for December 9, 2004 (P is measured in megawatts; t is measured in hours starting at midnight). Using the fact that power is the rate of change of energy, estimate the energy used on that day.

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66. On May 7, 1992, the space shuttle *Endeavour* was launched on mission STS-49, the purpose of which was to install a new perigee kick motor in an Intelsat communications satellite. The table gives the velocity data for the shuttle between liftoff and the jettisoning of the solid rocket boosters.