M E T U
Northern Cyprus Campus

	Calculus with Analytic Geometry **Short Exam 1**	

Code : *Math 119* Acad.Year: *2014-2015* Semester : *Fall* Date : *22.10.2014* Time : *18:45* Duration : *25 minutes*	Last Name: KEY Name: Signature: List No: Student No: 3 QUESTIONS 2 PAGES TOTAL 21 POINTS

1(9)	2(8)	3(4)			

Show your work! No calculators! Please draw a $\boxed{\text{box}}$ around your answers! Please do not write on your desk!

1. $(3 \times 3 = 9$ pts.$)$ Evaluate the limit, if it exists. **Give reasoning**.

DO NOT USE L'HOSPITAL'S RULE.

(a) $\displaystyle\lim_{x\to 1} \frac{|x-2|}{x^2+1} = \frac{\displaystyle\lim_{x\to 1} |x-2|}{\displaystyle\lim_{x\to 1} x^2+1} = \frac{|-1|}{2} = \frac{1}{2}$

$0 \neq \displaystyle\lim_{x\to 1} x^2+1$

(b) $\displaystyle\lim_{x\to 0} \frac{x^2+1}{x(x-2)^2}$ dne

$\quad 0 \quad 2$

$- \mid + \mid +$

$\displaystyle\lim_{x\to 0^-} \frac{x^2+1}{x(x-2)^2} = -\infty$

$\neq$

$\displaystyle\lim_{x\to 0^+} \frac{x^2+1}{x(x-2)^2} = +\infty$

(c) $\displaystyle\lim_{x\to -1} \frac{x+1}{\sqrt{x+2}-1} = \lim_{x\to -1} \frac{(x+1)(\sqrt{x+2}+1)}{(\sqrt{x+2}-1)(\sqrt{x+2}+1)}$

$\displaystyle = \lim_{x\to -1} \frac{(x+1)(\sqrt{x+2}+1)}{x+2-1} = \lim_{x\to -1} \sqrt{x+2}+1 = 2$

2. ($8 \times 1 = 8$ pts.) Determine whether the given statement is true or false.
Indicate your answers by typing **TRUE** or **FALSE**. No explanations required.

FALSE (a) If $\lim_{x\to a} f(x) = \infty$ and $\lim_{x\to a} g(x) = -\infty$, then $\lim_{x\to a}(f(x)+g(x)) = 0$.

FALSE (b) If $\lim_{x\to a} f(x)g(x)$ exists then $\lim_{x\to a} f(x)g(x) = f(a)g(a)$.

FALSE (c) If f is not differentiable at a then it is not continuous at a.

FALSE (d) If $\lim_{x\to a} f(x) = 0$ and $\lim_{x\to a} g(x) = 0$, then $\lim_{x\to a} \frac{f(x)}{g(x)}$ does not exist.

FALSE (e) $|x^2 - 2|$ is differentiable everywhere.

FALSE (f) $\lim_{x\to 0} \frac{1}{x^3} = +\infty$

TRUE (g) $\frac{x^2-1}{x-1}$ and $x+1$ are **NOT** the same function.

TRUE (h) $\lim_{x\to 2} \frac{x^2-1}{x-1} = \lim_{x\to 2} x+1$

3. ($1+1+1+1 = 4$ points) Fill in the blanks in the proof of the statement.

$$\lim_{x\to 2} -3x^2 + x + 3 = -7$$

Proof:
Given $\epsilon > 0$. We want to find $\delta > 0$ so that

$$0 < |x-2| < \delta \implies |(-3x^2+x+3) - (-7)| < \epsilon.$$

Now consider $|(-3x^2+x+3) - (-7)| = |-(x-2)(3x+5)| = |x-2||3x+5|$.
We want this quantity to be less than ϵ.
Suppose $\delta < 2$. Then if $|x-2| < \delta$,

$-2 < x-2 < 2 \Rightarrow 0 < x < 4 \Rightarrow 0 < 3x < 12$

$\Rightarrow 5 < 3x+5 < 17$

So we have $|3x+5| <$ 17.
So pick $\delta < \min($2, $\varepsilon/17)$.
Now, assume $0 < |x-2| < \delta$, so we have,
$|(-3x^2+x+3) - (-7)|$ $= |x-2||3x+5| < \delta \cdot 17 \le \frac{\varepsilon}{17} \cdot 17 = \varepsilon.$

$< \epsilon.$

This completes the proof.