M E T U
Northern Cyprus Campus

	Calculus with Analytic Geometry Short Exam 1	
Code : *Math 119* Acad.Year: *2012-2013* Semester : *Spring* Date : *13.03.2013* Time : *17:45* Duration : *40 minutes*	Last Name: Name: KEY Student No: Signature: 5 QUESTIONS ON 2 PAGES TOTAL 42+2=44 POINTS	

1	2	3	4	5			

Show your work! No calculators! Please draw a $\boxed{\text{box}}$ around your answers!
Please do not write on your desk!

1. (3 × 4 = 12 pts.) Evaluate the limit, if it exists. **Give reasoning.**

(a) $\lim\limits_{h\to 0} \dfrac{\frac{7}{x+h} - \frac{7}{x}}{h} = \lim\limits_{h\to 0} \dfrac{7x - 7(x+h)}{x(x+h)h} = \lim\limits_{h\to 0} \dfrac{-7h}{x(x+h)h}$

$= \lim\limits_{h\to 0} \dfrac{-7}{x(x+h)} = \dfrac{-7}{x^2}$

(b) $\lim\limits_{x\to 3} \dfrac{6}{3-x}$ d.n.e

$\lim\limits_{x\to 3^+} \dfrac{6}{3-x} = -\infty \qquad \lim\limits_{x\to 3^-} \dfrac{6}{3-x} = \infty$

(c) $\lim\limits_{x\to 2} \dfrac{x^3-8}{x+2} = \dfrac{\lim\limits_{x\to 2} x^3-8}{\lim\limits_{x\to 2} x+2} = \dfrac{0}{4} = 0$

2. (8 pts.) Find the number a which make $f(x)$ continuous. **Give reasoning.**

$$f(x) = \begin{cases} -2x^2 + 3x + a & \text{if } x \le 3 \\ \dfrac{3x^3 - 4x^2 - 14x - 3}{x-3} & \text{if } x > 3. \end{cases}$$

$$\begin{array}{l|l} 3x^3-4x^2-14x-3 & x-3 \\ \hline -(3x^3-9x^2) & 3x^2+5x+1 \\ \quad 5x^2-14x-3 & \\ \quad -(5x^2-15x) & \\ \qquad x-3 & \\ \qquad -(x-3) & \\ \qquad\quad 0 & \end{array}$$

$-2x^2+3x+a$ and $\dfrac{3x^3-4x^2-14x-3}{x-3}$ are cont., so f is cont. if it is cont at $x=3$.
To be cont at 3, we must have $f(3) = \lim\limits_{x\to 3} f(x)$

$f(3) = -9+a$

$\lim\limits_{x\to 3^-} f(x) = \lim\limits_{x\to 3^-} -2x^2+3x+a = -9+a$

$\lim\limits_{x\to 3^+} f(x) = \lim\limits_{x\to 3^+} \dfrac{3x^3-4x^2-14x-3}{x-3} = \lim\limits_{x\to 3^+} 3x^2+5x+1 = 43$

$\Rightarrow -9+a = 43 \Rightarrow \boxed{a=52}$

3. (a) (8 pts.) Show that $f'(1) = 6$ if $f(x) = 3x^2 + 5$, *using the limit definition of the derivative only.* (Note : Any other methods will not receive *any* credit.)

$$f'(1) = \lim_{h\to 0} \frac{f(1+h)-f(1)}{h} = \lim_{h\to 0} \frac{3(1+h)^2+5-8}{h}$$

$$= \lim_{h\to 0} \frac{\cancel{3}+6h+3h^2-\cancel{3}}{h} = \lim_{h\to 0} \frac{h(6+3h)}{h} = \lim_{h\to 0} 6+3h = 6.$$

(b) (4 pts.) Write the equation of the tangent line to the graph of $f(x) = 3x^2+5$ at the point $(1,8)$.

slope of tangent line is $f'(1)=6$ (by part a)

$\Rightarrow$ eqn : $y-8 = 6(x-1) \iff \boxed{y=6x+2}$

4. (10 pts.) Using the definition of the limit, prove that $\lim_{x\to 3} -2x^2+1 = -17$.

IMPORTANT NOTE about Question 4:

To receive 4 points automatically, leave the question unanswered and write "I do not know how to solve this question.".

Let $\varepsilon > 0$ be given. Want to find a $\delta > 0$ so that $0<|x-3|<\delta \implies |(-2x^2+1)-(-17)| < \varepsilon$.

Analyze $|(-2x^2+1)+17| = |-2x^2+18| = 2|x^2-9| = 2|x-3||x+3| < \varepsilon$ WANT!

Need a bound for $|x+3|$, so assume $|x-3| < \frac{1}{2}$

Then, $-\frac{1}{2} < x-3 < \frac{1}{2} \Rightarrow \frac{11}{2} < x+3 < \frac{13}{2} \Rightarrow |x+3| \overset{*}{<} \frac{13}{2}$

Choose $\delta < \min\{\frac{\varepsilon}{13}, \frac{1}{2}\}$, so $\delta \overset{**}{<} \frac{\varepsilon}{13}$ & $\delta \overset{*}{<} \frac{1}{2}$.

Assume that $0 < |x-3| \overset{**}{<} \delta$. Then

$$|(-2x^2+1)-(-17)| = 2|x-3|\cdot|x+3| \underset{*}{<} 2\cdot\frac{13}{2}\cdot|x-3| \underset{**}{<} 13\cdot\frac{\varepsilon}{13} = \varepsilon$$

So $|(-2x^2+1)-(-17)| < \varepsilon$ as required

5. (2 pts.) **Bonus Question** Write the schedule for any of the recitation sections for Math 119 this semester.

Mon / Wed 13:40–15:30

Tue 8:40–10:30

Thur 10:40–12:30