M E T U
Northern Cyprus Campus

	Calculus with Analytic Geometry Short Exam 2	
Code : *Math 119* Acad.Year: *2013-2014* Semester : *Fall* Date : *25.11.2013* Time : *17:45* Duration : *35 minutes*	Last Name: Name: KEY Student No: Signature:	
	3+1 QUESTIONS 2 PAGES TOTAL 20 + 2 POINTS	

1(5)	2(8)	3(7)	B(2)			

Show your work! No calculators! Please draw a $\boxed{\text{box}}$ around your answers!
Please do not write on your desk!

1. (5 pts.) Find the minimum distance of the parabola $x + y^2 = 0$ to the point $(0, -3)$.

Let (x,y) be a point on the parabola. Then the distance from the point $(0,-3)$ to (x,y) is: $d = \sqrt{x^2+(y+3)^2}$.

(x,y) is on the parabola, so $x=-y^2$. $\Rightarrow d = \sqrt{y^4+(y+3)^2}$.

Absolute extrema of d and $D = y^4+(y+3)^2$ occur at the same x value.

$D' = 4y^3 + 2y + 6 = (y+1)(4y^2-4y+6)$

	-1	
D'	$-$	$+$
D	$\searrow$	$\nearrow$

So absolute min. of D (so of d) is at $y=-1$.

Hence the min. distance from $(0,-3)$ to $x+y^2=1$ is

$\sqrt{(-1)^4+(-1+3)^2} = \sqrt{5}$

2. $(2+6 = 8$ pts.) Consider the function $f(x) = 3x^5 - 20x^3 - 2013^{119}$ on the interval $I = [-1, 10]$.

$= x^3(3x^2-20) - 2013^{119}$

(a) State the theorem that guarantees that $f(x)$ attains its absolute maximum and absolute minimum on I.

Extreme Value theorem: A continuous function f on a closed interval I has an absolute max. and absolute min.

(b) Find the above mentioned absolute maximum and minimum of $f(x)$ on I.

Abs. extrema may occur at critical pnts ($f'=0$ or f' dne) or end pnts.

$f'(x) = 15x^4 - 60x^2 = 15x^2(x-2)(x+2)$ (differentiable everywhere)

$f'(x) = 0 \iff x=0, \ x=2, \ x=-2 \longrightarrow$ not in I.

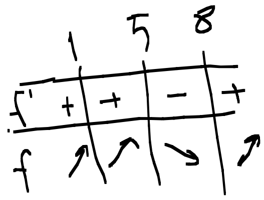
end pnts: $-1, 10$. (Let $2013^{119} = c$)

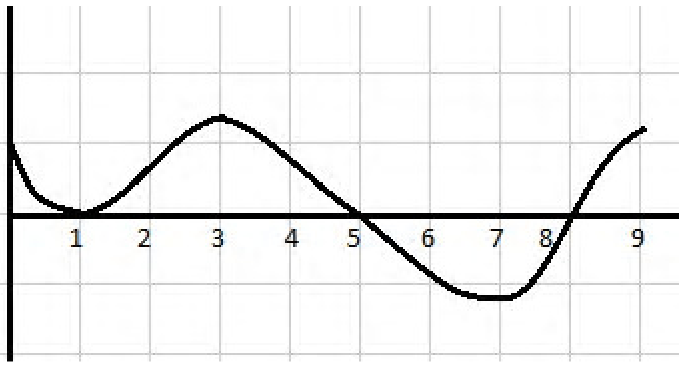
$f(-1) = 17-c \quad f(0) = -c \quad f(2) = 8(12-20)-c = -64-c$

$f(10) = 1000(300-20)-c = 280000-c$

So abs min is at 2 with value $-64-2013^{119}$ and
" max " " 10 " " $280000-2013^{119}$

3. $(7 \times 1 = 7$ pts.) Consider a function $f(x)$ whose **derivative** $f'(x)$ is given by the following figure.





(a) Find the intervals on which $f(x)$ is increasing. (f' should be positive)

$[0,1) \cup (1,5) \cup (8,9]$ (or $[0,5] \cup [8,9]$)

(b) Find the intervals on which $f(x)$ is decreasing. (f' should be negative)

$(5,8)$ (or $[5,8]$)

(c) Find the local maxima of $f(x)$.

at $x = 5$

(d) Find the local minima of $f(x)$.

at $x = 8$

(e) Find the intervals on which $f(x)$ is concave up. ($f'' > 0 \Rightarrow f'$ is increasing)

$(1,3) \cup (7,9)$

(f) Find the intervals on which $f(x)$ is concave down. ($f'' < 0 \Rightarrow f'$ is decreasing)

$(0,1) \cup (3,7)$

(g) Find the inflection points of $f(x)$. ($f'' = 0$ and concavity changes)

$x = 1, 3, 7$

4. Bonus $(1 + 0 + 1 = 2$ pts.) Determine whether the given statement is true or false. No explanations required.

FALSE (a) Only continuous functions have absolute extrema on a closed interval.

TRUE (b) I read all of the questions.

T/F (c) I read all of the questions **after** reading the question above.