Northern Cyprus Campus

	Calculus With Analytic Geometry Short Exam 1	
Code : *Math 119* Acad.Year: *2012-2013* Semester : *Fall* Date : *16.10.2012* Time : *17:45* Duration : *45 minutes*	Name: Last Name: Student No: Signature: 5 QUESTIONS ON 2 PAGES TOTAL 42 POINTS	

1	2	3	4	5			

Show your work! No calculators! Please draw a $\boxed{\text{box}}$ around your answers!
Please do not write on your desk!

1. (8 pts.) Evaluate the limit, if it exists. **Give reasoning.**

(a) $\displaystyle\lim_{x\to -2}\frac{x^2+5x+6}{x^2-x-6} = \lim_{\substack{x\to -2\\ x\neq -2}}\frac{(x+2)(x+3)}{(x+2)(x-3)}$

$\displaystyle = \lim_{x\to -2}\frac{x+3}{x-3} = \frac{-2+3}{-2-3} = \frac{1}{-5} = \boxed{-\frac{1}{5}}$

since $\frac{x+3}{x-3}$ is continuous at -2.

(b) $\displaystyle\lim_{x\to -1}\frac{x+1}{x^2-2x+1} = \frac{-1+1}{(-1)^2-2(-1)+1} = \frac{0}{1+2+1} = \boxed{0}$

since the function is continuous at -1.

(c) $\displaystyle\lim_{x\to 1}\frac{1}{x-1}$

$\displaystyle\lim_{x\to 1^-}\frac{1}{x-1} = -\infty$ (DNE) $\qquad \displaystyle\lim_{x\to 1^+}\frac{1}{x-1} = +\infty$ (DNE)

Overall $\displaystyle\lim_{x\to 1}\frac{1}{x-1}$ DNE.

2. (8 pts.) Find the number[redacted] a[redacted] which makes $f(x)$ [redacted] continuous. **Give reasoning.**

$$f(x) = \begin{cases} \dfrac{-5x^3-26x^2-22x+8}{x+4} & \text{if } x < -4 \\ 2x^2+2x+a & \text{if } x \geq -4. \end{cases}$$

Both pieces are continuous except possibly at endpoints.
For f to be continuous at -4,
we need $\displaystyle\lim_{x\to -4^-} f(x) = \lim_{x\to -4^+} f(x) = f(-4) = 2(-4)^2+2(-4)+a = 24+a$

$$\begin{array}{r|l} -5x^3-26x^2-22x+8 & x+4 \\ \cline{2-2} +5x^3+20x^2 & -5x^2-6x+2 \\ \hline -6x^2-22x & \\ +6x^2+24x & \\ \hline 2x+8 & \\ -2x\mp 8 & \\ \hline 0 & \end{array}$$

So $\displaystyle\lim_{x\to -4^-} f(x) = \lim_{x\to -4^-}(-5x^2-6x+2)$

$= -5(-4)^2-6(-4)+2 = -54$

$\Rightarrow \quad -54 = 24+a \Rightarrow \boxed{a=-78}$

3. (8 pts.) Find the following derivatives. **DO NOT SIMPLIFY YOUR ANSWERS.**

(a) $\frac{d}{dx}\left((3x-4x^3)(7+\underbrace{\sqrt{x}}_{x^{1/2}})\right) = (3-12x^2)\cdot(7+\sqrt{x}) + (3x-4x^3)\cdot(0+\frac{1}{2\sqrt{x}})$

(b) $\frac{d}{dt}\left(\frac{\sin(t)}{t^3+3}\right) = \frac{\cos t \cdot (t^3+3) - \sin t \cdot (3t^2)}{(t^3+3)^2}$

4. (8 pts.) Find $f'(3)$ if $f(x) = 3x^2 - 4x$, *using the limit definition of the derivative only.* (Note : Any other methods will not receive *any* credit.)

$$f'(3) = \lim_{h\to 0} \frac{f(3+h)-f(3)}{h} = \lim_{h\to 0} \frac{(3\cdot(3+h)^2-4(3+h)) - (3\cdot 3^2 - 4\cdot 3)}{h}$$

$$= \lim_{h\to 0} \frac{3(9+6h+h^2) - 12 - 4h - 3\cdot 9 + 4\cdot 3}{h}$$

$$= \lim_{h\to 0} \frac{\cancel{3\cdot 9} + 18h + 3h^2 \cancel{-12} - 4h - \cancel{3\cdot 9} + \cancel{12}}{h}$$

$$= \lim_{\substack{h\to 0\\ h\neq 0}} \frac{\cancel{h}(14+3h)}{\cancel{h}} = \lim_{h\to 0} 14 + 3h = \boxed{14}$$

5. (10 pts.) Using the definition of the limit, prove that $\lim_{x\to 3} 2x^2 - 1 = 17$.

Given $\varepsilon > 0$, want a δ so that $0 < |x-3| < \delta \Rightarrow |2x^2-1-17| < \varepsilon$.

Now, $|2x^2-1-17| = 2|x-3|\cdot|x+3|$

I can bound $|x-3|$, I need a new bound for $|x+3|$.

Assume $|x-3| < 1$ (Remember, I can freely impose conditions on $|x-3|$.)

Then $-1 < x-3 < 1 \Rightarrow 5 < x+3 < 7 \Rightarrow |x+3| < 7$. Pick $\delta < \min(1, \varepsilon/14)$

Now if $0 < |x-3| < \delta$, then

$$|(2x^2-1) - 17| = 2|x-3|\cdot|x+3| < 2\cdot\delta\cdot 7 = 14\cdot\frac{\varepsilon}{14} = \varepsilon.$$

Q.E.D.