## Mathematics Proficiency Exam

| $\begin{aligned} & \text { Acad.Year: 2011-2012 } \\ & \text { Semester : Fall } \\ & \text { Date : 21.9.2011 } \end{aligned}$ | Last Name: <br> Name |  |
| :---: | :---: | :---: |
| Time $: 9: 30$ Duration $: 120$ minutes | $\begin{gathered} \hline 30 \text { QUESTIONS ON } 10 \text { PAGES } \\ \text { TOTAL } 100 \text { POINTS } \\ \hline \end{gathered}$ |  |
|  | $S C O R E$ |  |

## Please show your work in all questions.

1. ( 3 pts ) What is the distance between the points $(1,4)$ and $(5,1)$ ?

$$
\text { distance }=
$$

2.(each 1.5 pts$)$

| Convert $\frac{9 \pi}{4}$ from radians to degrees. |  |
| :--- | :--- |

Convert $580^{\circ}$ from degrees to radians.
3. (each 1 pt$)$ A right triangle has hypotenuse of length 7 and legs of length 3 and $x$. If $\theta$ is the measure of the angle between the sides of length 7 and 3 , then


$$
\begin{array}{|l|l|}
\hline \sin (\theta)= \\
\hline
\end{array}
$$

$\square$
$\square$
$\square$

$$
\csc (\theta)=
$$

4. (each 1.5 pts$)$ Find the slope and $y$-intercept of the line through the points $(-1,2)$ and $(3,-4)$.

5. (3 pts) Write an equation for the circle centered at $(1,2)$ which passes through the point $(0,0)$.

6. (3 pts) Given two lines with the equations $y=\frac{1}{3} x+1$ and $x+3 y=2$, find the intersection point(s) if there is any.


## Name:

## Id Number:

7. (each 1.5 pts) Ayşe's money is 50 TL more than twice the money of Fatma. One third of Ayşe's money is 10TL less than Fatma's. How much money do they have?

| Ayse has: |
| :--- |
| Fatma has: |

8. (each 1.5 pts$)$ Determine the center and radius of the circle with the equation

$$
x^{2}-2 x+y^{2}+4 y-4=0
$$

| Center: |
| :--- |
| Radius: |

9. $(3 \mathrm{pts})$ Let $f(x)=\frac{1}{\sqrt{x+3}}$. Find the domain.

Domain:
10. (each 1.5 pts$)$ Find the domain and the range of $f(x)=|2 x-1|+5$.

## Domain: <br> Range:

11. (each 1 pt ) For the graph below, state the domain, the range, and the intervals on which the function is increasing, decreasing, or constant.


| domain: |
| :--- |
| range: |
| incr.: |
| decr.: |
| constant: |

12. (each 1 pt ) Use properties of functions to match each of the following functions with their graphs.

$f(x)=(x+3)^{2}-2$

| $f(x)=(x-3)^{2}-2$ |  |
| :--- | :--- |

$$
f(x)=2-(x+3)^{2}
$$

$$
f(x)=2-(x-3)^{2}
$$

## Name:

## Id Number:

13. (each 2 pts) Suppose that $f(x)=\sqrt{4 x-5}$ and $g(x)=x^{2}-3$.

For each function $h$ given below, find a formula for $h(x)$ and the domain of $h$.

| $h(x)=(f \circ g)(x)$ | $h(x)=$ | domain: |
| :--- | :--- | :--- |
| $h(x)=(g \circ f)(x)$ | $h(x)=$ | domain: |

14. ( 3 pts ) Find a formula for the inverse of the function $f(x)=\frac{x+2}{x-2}$.

$$
f^{-1}(x)
$$

15. (3 pts) Two ships start moving away from the same point at $t=0$, where $t$ denotes time. The first ship is sailing North, and the second ship is sailing East. The distance travelled by the first ship as a function of time is $t^{2}+2$ and the distance travelled by the second ship is $2 t+1$. Find the distance between the two ships at time $t$.

$$
D(t):
$$

16. $(3 \mathrm{pts})$ Find all real values of $x$ such that $x^{4}-2 x^{2}-3=0$.

$$
x=
$$

17. $(3 \mathrm{pts})$ Solve the inequality $\frac{(2 x+1)^{2}}{(x-10)(x+10)} \leq 0$. State the solution set using interval notation.
18. ( 3 pts ) Solve the inequality $x^{2}>x+2$. State the solution set using interval notation.
19. ( 3 pts ) Find the intersection points of the parabola $y=x^{2}+1$ with the line $y=13-x$.

| $x=$ |  |
| :--- | :--- |

20. $(3 \mathrm{pts})$ Calculate the product $(3-2 i) \cdot(5+7 i)$ where $i^{2}=-1$.
$\square$
21.(each 1 pt) Evaluate the following expressions. (Your answer may involve square roots.)

$$
\begin{array}{|l|}
\hline \sin \left(\frac{10 \pi}{3}\right)= \\
\hline \cos \left(\frac{10 \pi}{3}\right)= \\
\hline
\end{array}
$$

$$
\tan \left(\frac{10 \pi}{3}\right)=\square
$$

$$
\begin{array}{|l|l|}
\hline \cot \left(\frac{10 \pi}{3}\right)= \\
\hline
\end{array}
$$

| $\sec \left(\frac{10 \pi}{3}\right)=$ |
| :--- | :--- |

22. $\left(3\right.$ pts) Simplify the expression $\left[\frac{1}{\sec (x)-\tan (x)}+\frac{1}{\sec (x)+\tan (x)}\right]$ as much as possible.

23. (each 1.5 pts$)$ Find the exact numerical value of the compositions.

$$
\begin{array}{|c|l|}
\hline \tan \left(2 \arcsin \left(\frac{1}{2}\right)\right)= & \\
\hline \arctan (1)= & \\
\hline
\end{array}
$$

24. (each 1.5 pts$)$ Using the graph below, find the length of $B C$ and the area of the dashed region.


25. (each 1 pt ) Use properties of functions to match each of the following functions with their graphs.
A



| $f(x)=\tan (x)$ |  |
| :--- | :--- |

$$
f(x)=\cos \left(\frac{1}{2} x\right)+1
$$


D


$$
\begin{array}{|l|l|}
\hline f(x)=\cos (2 x)+1 \\
\hline f(x)=2 \sin (x) \cos (x) \\
\hline
\end{array}
$$

26. (each 1.5 pts$)$ Find the exact numerical value of the following expressions.

$$
\begin{array}{|l|}
\hline 3 \ln \left(e^{7}\right)= \\
\hline \\
\hline \log _{2} \sqrt{64}= \\
\hline
\end{array}
$$

27. (3 pts) Find the domain of the function $f(x)=\frac{1}{\log _{3}(2-x)}$.

28. (3 pts) Find the solution to the exponential equation $7\left(2^{t}\right)=5\left(8^{t}\right)$.

| $t=$ |  |
| :--- | :--- |

29. (3 pts) Find the exact numerical value of the expression $2 \sin \left(\frac{\pi}{8}\right) \cos \left(\frac{\pi}{8}\right)$.

30. (each 1 pt$)$ Match each function below with its graph.


