METU - NCC

	Precalculus Final	
Code : *Math 100* Acad.Year : *2013-2014* Semester : *Spring* Date : *4.6.2014* Time : *9:00* Duration : *100 minutes*	Last Name: Name : KEY Department: Signature: 10 QUESTIONS ON 4 PAGES TOTAL 100 POINTS	Student No.: Section:

1 (8)	2 (10)	3 (8)	4 (8)	5 (10)	6 (12)	7 (12)	8 (10)	9 (10)	10 (12)			

1. *(8 pts)* Write the equations of the following lines through the point $(5, 7)$.

(a) Whose slope is 2.

Line eqn: $y = mx + b$ where $m = 2$ is given

so, $y = 2x + b$

$7 = 2 \cdot 5 + b \Rightarrow b = -3 \Rightarrow y = 2x - 3$

(b) Whose x-intercept is 2.

Line eqn passing through $(x_1, y_1), (x_2, y_2)$ is: $\frac{y - y_1}{x - x_1} = \frac{y_2 - y_1}{x_2 - x_1}$

so, $\frac{y - 0}{x - 2} = \frac{7 - 0}{5 - 2} \Rightarrow y = \frac{7}{3}(x - 2)$ or $y = \frac{7}{3}x - \frac{14}{3}$

2. *(10 pts)* Solve each equation.

(a) $4^x - 2^{x+1} = 8$

Say $2^x = a$ then eqn becomes, $a^2 - 2a - 8 = 0$

$(a - 4)(a + 2) = 0$

$a = 4 \Rightarrow 2^x = 4$

$x = 2$

$a = -2 \Rightarrow 2^x = -2$ — no solution.

$\boxed{x = 2}$

(b) $(\ln x)^3 = \ln(x^4)$

Say $\ln x = a$ then eqn becomes, $a^3 = 4a$

$a^3 - 4a = 0$

$a(a - 2)(a + 2) = 0$.

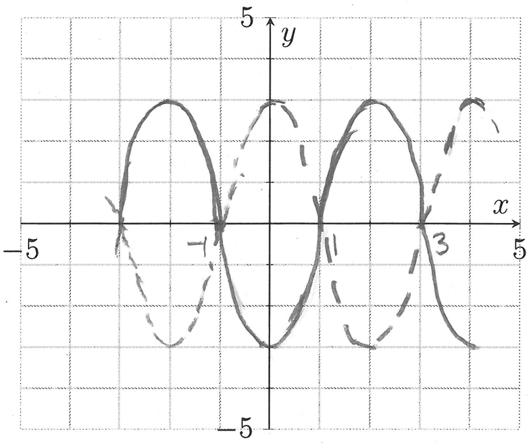
$a = 0 \Rightarrow \ln x = 0$, $x = 1$

$a = 2 \Rightarrow \ln x = 2$, $x = e^2$

$a = -2 \Rightarrow \ln x = -2$, $x = e^{-2}$

$\boxed{x = 1, e^2, e^{-2}}$

3. *(8 pts)* Sketch the graph of $f(x) = A\cos(Bx + C)$ for $-5 \le x \le 5$ where the period of function $f(x)$ is 4, amplitude is 3 and it passes through the point $(-1, 0)$.



$A = 3$

$\frac{2\pi}{B} = 4 \Rightarrow B = \frac{\pi}{2}$

$\Rightarrow 3\cos\left(\frac{\pi}{2}\cdot -1 + C\right) = 0$

$\Rightarrow C - \frac{\pi}{2} = \frac{\pi}{2}$ or $C - \frac{\pi}{2} = \frac{3\pi}{2}$

$C = \pi$ or $C = 2\pi$

4. *(8 pts)* Find the following values.

(a) $\cos(\frac{63\pi}{14}) = \cos\left(\frac{9\pi}{2}\right)$

$= \cos\left(\frac{\pi}{2}\right)$

$= 0$

$\tan(\frac{63\pi}{12}) = \tan\left(\frac{21\pi}{4}\right)$

$= \tan\left(\frac{\pi}{4}\right)$

$= 1$

(b) $\sin(-225°) = -\sin 45°$

$= -\frac{\sqrt{2}}{2}$

$\cot(-225°) = \cot 45°$

$= 1$

5. *(10 pts)* Compute the following values by using related trigonometric formulas.

(a) $\cos(-112.5°) = \cos\left(-\frac{225°}{2}\right) = \cos\left(\frac{225}{2}\right)$

$= \sqrt{\frac{1 - \frac{\sqrt{2}}{2}}{2}}$ (using half-angle formula)

$\boxed{\frac{\sqrt{2-\sqrt{2}}}{2}}$

(b) $\tan(165°) = -\tan 15°$

$= -\sqrt{\frac{1 - \frac{\sqrt{3}}{2}}{1 + \frac{\sqrt{3}}{2}}}$ (using half-angle formula)

$\boxed{\frac{\sqrt{2-\sqrt{2}}}{2}}$

6. *(12 pts)* Solve the following trigonometric equation:

$$\frac{1-\sin 2x}{\cos 2x} = \frac{1-\tan x}{1+\tan x}$$

$$\frac{1-\sin 2x}{\cos 2x} = \frac{1-2\sin x\cos x}{\cos^2 x-\sin^2 x} = \frac{\sin^2 x+\cos^2 x-2\sin x\cos x}{(\cos x-\sin x)(\cos x+\sin x)}$$

$$= \frac{(\cos x-\sin x)^{\cancel{2}}}{\cancel{(\cos x-\sin x)}(\cos x+\sin x)} = \frac{\cos x-\sin x}{\cos x+\sin x}$$

$$= \frac{\frac{\cos x}{\cos x}-\frac{\sin x}{\cos x}}{\frac{\cos x}{\cos x}+\frac{\sin x}{\cos x}} = \frac{1-\tan x}{1+\tan x}$$

$$\boxed{x = \mathbb{R} - \left\{\left\{\frac{\pi}{4}+\frac{k\pi}{2}\right\} \cup \left\{\frac{\pi}{2}+k\pi\right\}\right\}}$$

We have to exclude such values!

$\cos 2x = 0$; $\tan x = -1$; $\tan x = \mp\infty$

7. *(12 pts)* If $\cos x = a$ then use basic trigonometric identities to write $\cos 4x$ in terms of a.

$$\cos 2x = 2\cos^2 x - 1 = 2a^2 - 1$$

$$\cos 4x = 2\cos^2(2x) - 1 = 2(2a^2-1)^2 - 1$$

8. *(10 pts)* Find $\tan y$ if we know that $\tan x = 3$ and $\tan(x+y) = 33$

$$\tan(x+y) = \frac{\tan x + \tan y}{1 - \tan x \tan y} \Rightarrow 33 = \frac{3 + \tan y}{1 - 3 \cdot \tan y}$$

$$\Rightarrow 33 - 99 \tan y = 3 + \tan y \Rightarrow 100 \tan y = 30$$

$$\Rightarrow \tan y = \frac{3}{10}$$

9. *(10 pts)* Determine whether the following triangles are possible or impossible by using law of sine and/or law of cosine.

(a) Sides of the triangle ABC: $a = 8$, $b = 11$, $c = 13$ and angle: $\hat{A} = 100°$

Cosine Law: $a^2 = b^2 + c^2 - 2bc \cos \hat{A}$

$$8^2 = 11^2 + 13^2 - 2.11.13. \cos \hat{A}$$

$$\Rightarrow \cos \hat{A} = \frac{121 + 169 - 64}{2 \cdot 11 \cdot 13} > 0$$

$\hat{A}$ is in the second quadrand, $\cos \hat{A}$ must be negative. Impossible.

(b) Sides of the triangle ABC: $a = 5$, $b = 10$ and angles: $\hat{A} = 30°$, $\hat{B} = 90°$

Sine Law: $\frac{a}{\sin \hat{A}} = \frac{b}{\sin \hat{B}}$

$$\frac{5}{\sin 30°} = \frac{10}{\sin 90°}$$

$$\frac{5}{\frac{1}{2}} = \frac{10}{1} \quad \checkmark \text{ possible.}$$

10. *(12 pts)* Solve the following trigonometric equation for $x \in [0, 2\pi]$:

$$\sin(x) + \cos(x) = 1$$

By taking square of both sides;

$$\underbrace{\sin^2 x + \cos^2 x}_{1} + 2 \sin x \cos x = 1$$

$$\Rightarrow 2 \sin x \cos x = 0 \Rightarrow \sin x = 0 \quad ; \quad \cos x = 0$$

$$x = 0, \pi, 2\pi \quad ; \quad x = \frac{\pi}{2}, \frac{3\pi}{2}$$

Check all solutions!

Exclude $\frac{3\pi}{2}$ and π

$$\boxed{x = 0, \frac{\pi}{2}, 2\pi}$$