METU - NCC

	Precalculus Midterm	
Code : *Math 100* Acad.Year: *2012-2013* Semester : *Fall* Date : *29.11.2012* Time : *17:40* Duration : *100 minutes*	Last Name: Name : Department: Signature:	Student No.: Section:
	10 QUESTIONS ON 4 PAGES TOTAL 100 POINTS	

1 (9)	2 (9)	3 (9)	4 (9)	5 (9)	6 (9)	7 (9)	8 (16)	9 (9)	10 (12)			

1. *(9 pts)* Find all x satisfying $|2x+4| - x > 6$.

When $2x+4 > 0$ $(\Rightarrow x > -2)$;

$2x+4-x > 6 \Rightarrow x > 2$

When $2x+4 < 0$ $(\Rightarrow x < -2)$;

$-2x-4-x > 6 \Rightarrow x < -\frac{10}{3}$

When $2x+4 = 0$ $(\Leftrightarrow x = -2)$

$0-(-2) > 6$ is not correct.

$(-\infty, -\frac{10}{3}) \cup (2, \infty)$

2. *(9 pts)* The sum of squares of three consecutive integers equals 10 times the largest of these three consecutive integers. What is the smallest one?

Let's say the numbers are: $n, n+1, n+2$. So our equation becomes; $n^2+(n+1)^2+(n+2)^2 = 10(n+2)$

$\Rightarrow n^2+n^2+2n+1+n^2+4n+4 = 10n+20$

$\Rightarrow 3n^2-4n-15 = 0$

$(3n+5)(n-3) = 0 \Rightarrow n = 3$ or ~~$n = -5/3$~~ (not an integer!)

the smallest $= 3$

3. *(9 pts)* Find all solutions of x satisfying $x^4+3x^2+4 = 16x^2-32$.

$x^4+3x^2+4 = 16x^2-32$

$\Rightarrow x^4-13x^2+36 = 0$

$\Rightarrow (x^2-9)(x^2-4) = 0$

$\Rightarrow (x-3)(x+3)(x-2)(x+2) = 0$

Solutions: $\{-3, -2, 2, 3\}$

4. *(9 pts)* Let $P(x) = x^3 + 3x^2 + 2x$. Evaluate $P(-1+i)$. Simplify the result as much as possible.

$P(x) = x^3+3x^2+2x = x(x^2+3x+2) = x(x+1)(x+2)$

$\Rightarrow P(-1+i) = (-1+i)(-1+i+1)(-1+i+2)$

$= (-1+i)(i)(1+i)$

$= i\cdot(i^2-1)$

$= i\cdot(-1-1)$

$= -2i$

Result $= -2i$

5. *(9 pts)* Divide $P(x) = x^4 - 4x^2 + 2x + 4$ by $Q(x) = x - 3$ and find the remainder.

$x^4-4x^2+2x+4 \;\big|\; x-3$

$-x^4+3x^3$

$3x^3-4x^2+2x+4$

$-3x^3+9x^2$

$5x^2+2x+4$

$-5x^2+15x$

$17x+4$

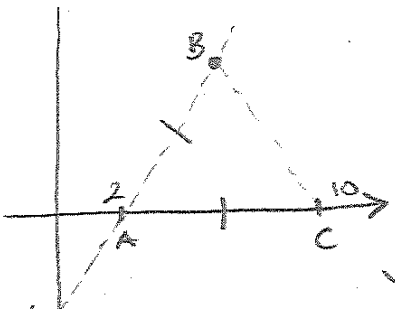
$-17x+51$

55

$x^3+3x^2+5x+17$

Remainder $= 55$

6. *(9 pts)* Find the coordinates of the point B=(x, y) on the line $y = 2x-4$ so that AB=AC; where A = $(2, 0)$ and C= $(10, 0)$. That is, the triangle ABC is isosceles (ikizkenar).



$d_{AC} = 10-2 = 8$. Coordinates of B: $(x, 2x-4)$

$d_{AB} = \sqrt{(x-2)^2+(2x-4-0)^2}$

since $d_{AB} = d_{AC}$

$\sqrt{(x-2)^2+(2x-4)^2} = 8$

$x = 2+\frac{8}{\sqrt{5}}$ or $x = 2-\frac{8}{\sqrt{5}}$

$B\left(2+\frac{8}{\sqrt{5}}, \frac{16}{\sqrt{5}}\right)$ or $B\left(2-\frac{8}{\sqrt{5}}, -\frac{16}{\sqrt{5}}\right)$

7. *(7 pts)* Find an equation of the line passing through A(-1,-2) and B(-4,-21).

Line eqn: $\frac{y-(-21)}{x-(-4)} = \frac{(-2)-(-21)}{(-1)-(-4)} \Rightarrow y = \frac{19}{3}x + \frac{13}{3}$

$y = \frac{19}{3}x + \frac{13}{3}$

8. *(2x8 pts)* Complete the table below by filling in correct values of $(f \circ f)(x)$ and $(g \circ f^{-1})(x)$:

x	1	2	3	4
$f(x)$	2	3	4	1
$g(x)$	2	2	1	3
$(f \circ f)(x)$	3	4	1	2
$(g \circ f^{-1})(x)$	3	2	2	1

9. *(9 pts)* Find the domain of the function $f(x) = \frac{1}{\sqrt[3]{1-x^2}-1}$.

Roots of the denominator should be excluded.

$\sqrt[3]{1-x^2} - 1 = 0$

$\Rightarrow \sqrt[3]{1-x^2} = 1$

$\Rightarrow 1 - x^2 = 1 \Rightarrow x = 0$

Domain $f(x) = \mathbb{R} - \{0\}$

10. *(16 pts)* The graph of $f(x)$ is given below. Graph $g(x) = 2 - f(-x-1)$ in the blank grid. Show individual steps of the transformation of the graph of $f(x)$ to the graph of $g(x)$.