M E T U
Northern Cyprus Campus

	Calculus for Functions of Several Variables Short Exam 2	
Code : *Math 120* Acad.Year: *2013-2014* Semester : *Fall* Date : *25.11.2013* Time : *17:45* Duration : *35 minutes*	Last Name: Name: Student No: Signature: 4+1 QUESTIONS ON 2 PAGES TOTAL 20 + 2 POINTS	

1(8)	2(8)	3(2)	4(2)	B(2)	KEY		

Show your work! No calculators! Please draw a $\boxed{\text{box}}$ around your answers!

Please do not write on your desk!

1. $(2+2+2+2 = 8$ pts.) Let $f(x,y) = 2x^2 + 3y^2 - 4x - 5$ and let $D = \{(x,y)x^2 + y^2 \leq 16\}$.

(a) Find and classify all the local extrema of $f(x,y)$ inside $\{(x,y)|\, x^2+y^2 < 16\}$.

$\nabla f = \langle 4x-4, 6y\rangle = \langle 0,0\rangle \Rightarrow x=1, y=0 \Rightarrow (1,0) \in \{x^2+y^2<16\}$

interior of D does not have a boundary

f is a polynomial so everywhere differentiable.

$f_{xx}(1,0) = 4 \quad f_{xy}(1,0) = 0 \quad (f_{xx}f_{yy} - f_{xy}^2)(1,0) = 24 > 0$

$f_{yx}(1,0) = 0 \quad f_{yy}(1,0) = 6 \quad f_{xx}(1,0) = 4 > 0$

f has a local minimum at (1,0) with value −7.

(b) State the theorem that guarantees the existence of maxima and minima on D.

Suppose f is continuous on a closed bounded domain

Then f attains its absolute maximum and minimum.

(c) Use The **Method of Lagrange Multipliers** to find the maximum and minimum of $f(x,y)$ on the boundary of $D = \{(x,y)|\, x^2+y^2 = 16\}$.

$\begin{Bmatrix} \nabla f = \lambda \cdot \nabla g \\ g(x,y) = 16 \end{Bmatrix} \equiv \begin{Bmatrix} 4x-4 = \lambda\cdot 2x \\ 6y = \lambda \cdot 2y \\ x^2+y^2 = 16 \end{Bmatrix}$ $\quad g(x,y) \quad 2y(\lambda-3) = 0$

$y=0 \Rightarrow x = \mp 4$. $\lambda = 3 \Rightarrow 4x-4 = 6x \Rightarrow x = -2 \Rightarrow y = \pm 2\sqrt{3}$

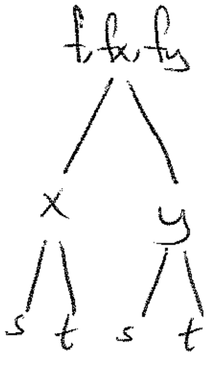
$\underbrace{f(-4,0) = 11}_{\text{min}}, f(4,0) = 43 \quad \underbrace{f(-2,2\sqrt{3}) = f(-2,-2\sqrt{3}) = 47}_{\text{max}}$

(d) Now that we are sure that the maximum and minimum of $f(x,y)$ is attained on D, find these extrema and the points that they are attained.

Absolute max. of f on D is 47 attained at $(-2, \pm 2\sqrt{3})$.

Absolute min. of f on D is −7 attained at (1,0).

2. (3+3+2=8) Find the following partial derivatives for the differentiable function $f(x, y)$ where $x = s^2 + t^2$ and $y = st$. You can use the symbols f_x and f_y to simplify your answer.



(a) $\dfrac{\partial f}{\partial s} = f_x \cdot x_s + f_y \cdot y_s$

$= f_x \cdot 2s + f_y \cdot t$

(b) $\dfrac{\partial f}{\partial t} = f_x \cdot x_t + f_y \cdot y_t$

$= f_x \cdot 2t + f_y \cdot s$

(c) $\dfrac{\partial}{\partial t}\dfrac{\partial f}{\partial t} = \dfrac{\partial}{\partial t}\left(f_x \cdot 2t + f_y \cdot s \right)$

$t - x - y - t \quad y_t \quad t \quad$ no t.

$= \left[\dfrac{\partial}{\partial t}(f_x) \cdot 2t + f_x \cdot \dfrac{d}{dt}(2t)\right] + \left(\dfrac{\partial}{\partial t} \cdot f_y\right) \cdot s$

$= \left[\left[f_{xx} \cdot \underbrace{x_t}_{4t} + f_{xy} \cdot \underbrace{y_t}_{s} \right] \cdot 2t + f_x \cdot 2 + \left[f_{yx} \cdot \underbrace{x_t}_{4t} + f_{yy} \cdot \underbrace{y_t}_{s} \right] \cdot s\right.$

$= f_{xx} \cdot 8t^2 + f_{xy} \cdot 2st + 2 f_x + f_{yx} \cdot 4st + f_{yy}\, s^2$

3. (2 pts.) Find an equation of the tangent plane to the surface $z = \underbrace{3^x + e^y}_{f(x,y)}$ at the point $(-1, 0)$.

$f_x = 3^x \cdot \ln 3 \qquad f_y = e^y.$

eqn. of the tgt. plane: $z = (3^{-1} + e^0) + f_x(-1,0) \cdot (x+1) + f_y(-1,0) \cdot (y-0)$

$\boxed{z = \dfrac{4}{3} + \dfrac{\ln 3}{3}(x+1) + y}$

4. (2 pts.) Find **a normal vector** to the tangent plane of the hyperboloid of two sheets $\underbrace{x^2 - 3y^2 + 5z^2}_{g(x,y,z)} = 6$ at the point $(-2, 1, 1)$.

$n = \nabla g(-2,1,1)$

$n = \nabla g(-2,1,1) = \langle 2x, -6y, 10z \rangle \Big|_{(-2,1,1)} = \langle -4, -6, 10 \rangle$

5. Bonus $(1 + 0 + 1 = 2$ pts.) Determine whether the given statement is true or false. No explanations required.

F (a) Only continuous functions have absolute extrema on a closed bounded domain. F

F/T (b) I read all of the questions. T/F

T/F (c) I read all of the questions after reading the question above. F/T