M E T U
Northern Cyprus Campus

	Calculus for Functions of Several Variables Short Exam 1	
Code : *Math 120* Acad.Year : *2011-2012* Semester : *Summer* Date : *18.7.2012* Time : *18:40* Duration : *35 minutes*	Last Name: Department: Section: Recitation: 5 QUESTIONS ON 2 PAGES TOTAL 45 POINTS	Name: Student No: Signature:

1	2	3	4	5	6	ABOZER		

Show your work! No calculators! Please draw a $\boxed{\text{box}}$ around your answers!
Please do not write on your desk!

1. (1 pt.) What is the internet address of the Mathematics Group of METU NCC?

math.ncc.metu.edu.tr

2. ($4 \times 4 = 16$ pts.) Find the limit, if it exists and prove your claim. Otherwise, show that the limit does not exist.

(a) $\displaystyle \lim_{(x,y)\to(0,0)} \frac{x^3y}{x^4+y^4}$

$$\lim_{\substack{(x,y)\to(0,0)\\ x=0}} \frac{x^3y}{x^4+y^4} = \lim_{(x,y)\to(0,0)} \frac{0}{y^4} = 0 \quad \Big| \quad \lim_{\substack{(x,y)\to(0,0)\\ y=x}} \frac{x^3y}{x^4+y^4} = \lim_{(x,y)\to(0,0)} \frac{x^4}{2x^4} = \frac{1}{2}$$

Hence this limit does not exist.

(b) $\displaystyle \lim_{(x,y)\to(0,0)} \frac{\cos^2(x^2+\frac{\pi}{2})\,y^5}{x^4+3y^4}$

$$0 \leqslant \left| \cos^2(x^2+\tfrac{\pi}{2}) \cdot y \cdot \frac{y^4}{x^4+3y^4} \right| \leqslant \cos^2(x^2+\tfrac{\pi}{2}) \cdot |y| \leqslant |y|$$

$\displaystyle \lim_{(x,y)\to(0,0)} |y| = 0$. & $\displaystyle \lim_{(x,y)\to(0,0)} -|y| = 0$

$$\Rightarrow -|y| \leqslant \frac{\cos^2(x^2+\frac{\pi}{2})\,y^5}{x^4+3y^4} \leqslant |y|$$

By Squeeze Theorem $\displaystyle \lim_{(x,y)\to(0,0)} \frac{\cos^2(x^2+\frac{\pi}{2})\,y^5}{x^4+3y^4} = 0$

(c) $\displaystyle \lim_{(x,y)\to(0,0)} \frac{e^{x^2}}{x^2+y^2}$

$$\left.\begin{array}{l} \displaystyle\lim_{(x,y)\to(0,0)} e^{x^2} = 1 \\ \displaystyle\lim_{(x,y)\to(0,0)} x^2+y^2 = 0 \end{array}\right\}$$ limit does not exist.

(d) $\displaystyle \lim_{(x,y)\to(0,0)} \frac{\sin(x^2)\cos(y^4)+\sin(y^4)\cos(x^2)}{x^2+y^4} = \lim_{(x,y)\to(0,0)} \frac{\sin(x^2+y^4)}{x^2+y^4} = 1$

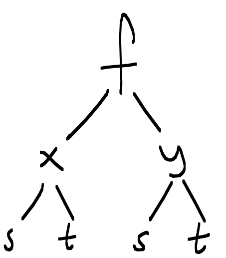
3. (3 +3 = 6 pts.) Find the following partial derivatives for the differentiable function

$$f(x,y) = \int_1^{xy^2} \cos(1 - t + t^2)\, dt$$

(a) $\frac{\partial f}{\partial x} = \cos\left(1 - xy^2 + x^2y^4\right) \cdot \frac{\partial}{\partial x}\left(xy^2\right) = \cos\left(1 - xy^2 + x^2y^4\right) \cdot y^2$

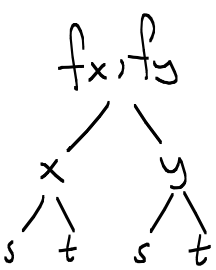
(b) $\frac{\partial f}{\partial y} = \cos\left(1 - xy^2 + x^2y^4\right) \cdot \frac{\partial}{\partial y}\left(xy^2\right) = \cos\left(1 - xy^2 + x^2y^4\right) \cdot 2xy$

4. (4+4+8=16) Find the following partial derivatives for the differentiable function $f(x,y)$ where $x = st$ and $y = s^2 + t^2$.



(a) $\frac{\partial f}{\partial s} = f_x \frac{\partial x}{\partial s} + f_y \frac{\partial y}{\partial s} = f_x \cdot t + f_y \cdot 2s$

(b) $\frac{\partial f}{\partial t} = f_x \cdot \frac{\partial x}{\partial t} + f_y \cdot \frac{\partial y}{\partial t} = f_x \cdot s + f_y \cdot 2t$



(c) $\frac{\partial}{\partial t}\frac{\partial f}{\partial s} = \frac{\partial}{\partial t}\left(f_x \cdot t + f_y \cdot 2s\right)$

$= \left[\frac{\partial f_x}{\partial t} \cdot t + f_x \cdot 1\right] + \frac{\partial f_y}{\partial t} 2s =$

$= \left[\left(f_{xx} \cdot s + f_{xy} \cdot 2t\right) \cdot t + f_x\right] + \left(f_{yx} \cdot s + f_{yy} \cdot 2t\right) \cdot 2s$

5. (4 pts.) Find the equation of the tangent plane to the surface $z = e^x + y^2$ at the point $(1,1)$. $z_0 = e + 1$

$\left.\frac{\partial z}{\partial x}\right|_{(1,1)} = \left. e^x \right|_{(1,1)} = e$ $\quad\Big\}\quad$ $\left.\frac{\partial z}{\partial y}\right|_{(1,1)} = \left. 2y \right|_{(1,1)} = 2$

Tangent line eqn: $z = e+1 + e(x-1) + 2(y-1)$

6. (2 pt.) Because of scheduling conflicts, we have to make-up Math 120 on the designated date 21.7.2012. Please answer the following questions with a **Y**(es) or **N**(o).

(a) Are you available on 21.7.2012 between the hours of 09:40 and 12:30? Y

(b) Are you available on 21.7.2012 between the hours of 14:40 and 17:30? Y