M E T U
Northern Cyprus Campus

	Calculus for Functions of Several Variables Short Exam 2	

Code : *Math 120* Acad.Year: *2011-2012* Semester : *Spring* Date : *16.4.2012* Time : *17:45* Duration : *45 minutes*	Last Name: Department: Section: Recitation: 4 QUESTIONS ON 4 PAGES TOTAL 50 POINTS	Name: Student No: Signature:

1	2	3	4			

Show your work! No calculators! Please draw a $\boxed{\text{box}}$ around your answers!
Please do not write on your desk!

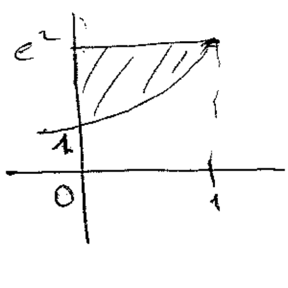
1. ($2 \times 8 = 16$ pts.) Let D be the region in the 2−dimensional space which is bounded by the curves $y = e^{2x}$, $y = e^2$, $x = 0$, and $x = 1$. Let $f(x, y)$ be a function on D.

(a) Write $\iint_D f(x, y)\, dA$ as an iterated integral with dx as the outermost integration variable. That is, find $\alpha, \beta, \gamma, \theta$ so that

$$\iint_D f(x, y)\, dA = \int_\alpha^\beta \int_\gamma^\theta f(x, y)\, dy\, dx$$

$\alpha =$ 0 ; $\beta =$ 1 ; $\gamma =$ e^{2x} ; $\theta =$ e^2

DO NOT EVALUATE THIS INTEGRAL.



$$\int_0^1 \int_{e^{2x}}^{e^2} f(x,y)\, dy\, dx$$

$$\int_1^{e^2} \int_0^{\frac{\ln y}{2}} f(x,y)\, dx\, dy$$

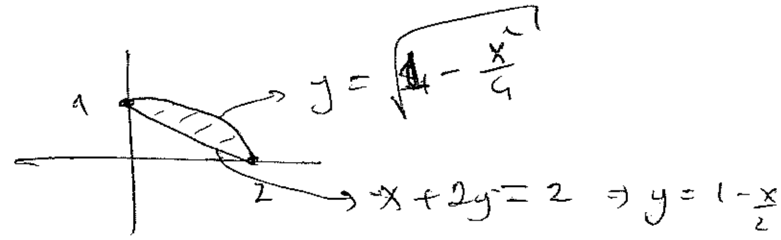
$y = e^{2x}$
$\Rightarrow \ln y = 2x$

(b) Write $\iint_D f(x, y)\, dA$ as an iterated integral with dy as the outermost integration variable. That is, find $\alpha, \beta, \gamma, \theta$ so that

$$\iint_D f(x, y)\, dA = \int_\alpha^\beta \int_\gamma^\theta f(x, y)\, dx\, dy$$

$\alpha =$ 1 ; $\beta =$ e^2 ; $\gamma =$ 0 ; $\theta =$ $\frac{\ln y}{2}$

DO NOT EVALUATE THIS INTEGRAL.



2. ($4 + 12 = 16$ pts.) Let R be the region in the 2−dimensional space which is in the first quadrant and bounded by the ellipse $x^2 + 4y^2 = 4$. and the line through points $(0, 1)$ and $(2, 0)$. Let $f(x, y)$ be a function that is defined on R.

$$\frac{x^2}{2^2} + \frac{y^2}{1^2} = 1$$

(a) Write $\iint_R f(x, y)\, dA$ as an iterated integral in Cartesian coordinates. DO NOT EVALUATE THIS INTEGRAL.

$$\int_0^2 \int_{1-\frac{x}{2}}^{\sqrt{1-\frac{x^2}{4}}} f(x,y)\, dy\, dx$$

$$\int_1^2 \int_{2-2y}^{\sqrt{4-4y^2}} f(x,y)\, dx\, dy$$

OK

(b) Write $\iint_R f(x, y)\, dA$ as an iterated integral in polar coordinates. DO NOT EVALUATE THIS INTEGRAL.

$x = r\cos\theta$
$y = r\sin\theta$

$$x^2 + 4y^2 = 4$$

$$\Leftrightarrow\ r^2(\cos^2\theta + 4\sin^2\theta) = 4$$

$= 1 + 3\sin^2\theta$

$$r = \sqrt{\frac{4}{1+3\sin^2\theta}}$$

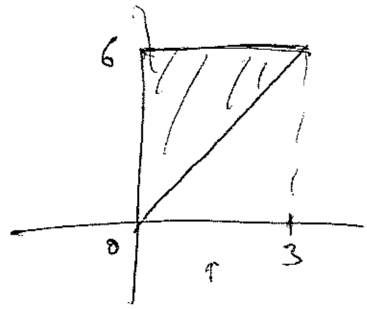
$$x + 2y = 2$$

$$\Leftrightarrow\ r(\cos\theta + 2\sin\theta) = 2$$

$$r = \frac{2}{\cos\theta + 2\sin\theta}$$

$$\int_0^{\pi/2} \int_{2/(\cos\theta + 2\sin\theta)}^{2\sqrt{\frac{1}{1+3\sin^2\theta}}} f(r\cos\theta, r\sin\theta)\, r\, dr\, d\theta$$

3. ($4 + 6 = 10$ pts.) Consider the integral $\int_0^3 \int_{2x}^6 e^{4x/y}\, dy\, dx$



(a) Change the order of integration.

$$I = \int_0^6 \int_0^{y/2} e^{4x/y}\, dx\, dy$$

(b) Evaluate the integral

$$\frac{4}{\cancel{y}} \frac{\cancel{y}}{2}$$

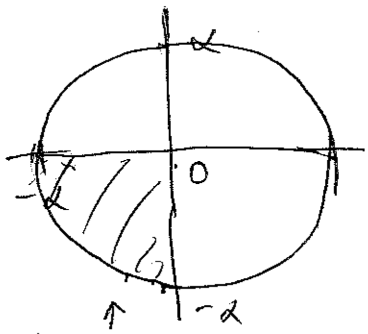
$$I = \int_0^6 \frac{y}{4} e^{4x/y} \Big|_0^{y/2} dy$$

$$= \int_0^6 \frac{y}{4} (1 - e^2)\, dy$$

$$= \frac{1-e^2}{4} \left(\frac{y^2}{2} \Big|_0^6 \right) = \frac{1-e^2}{4} \cdot 18 = 9 \frac{1-e^2}{2}$$

$\alpha > 0$

4. (8 pts.) Use polar coordinates to evaluate $\displaystyle\int_{-a}^{0}\int_{-\sqrt{a^2-x^2}}^{0} e^{x^2+y^2}\,dy\,dx$



$y = -\sqrt{\alpha^2 - x^2}$

$\Rightarrow y^2 + x^2 = \alpha^2$

$$\int_{\pi}^{3\pi/2}\int_{0}^{\alpha} e^{r^2}\,\frac{2r}{2}\,dr\,d\theta$$

$$= \int_{\pi}^{3\pi/2} \frac{e^{r^2}}{2}\Bigg|_{r=0}^{\alpha} d\theta$$

$$= \frac{e^{\alpha^2}-1}{2}\cdot\theta\Bigg|_{\theta=\pi}^{3\pi/2} = \frac{e^{\alpha^2}-1}{2}\left(\frac{\pi}{2}\right)$$

$$= \pi\,\frac{e^{\alpha^2}-1}{4}$$