

## Determinants:

1. Find  $A^{-1}$  using the adjoint matrix:

$$A = \begin{bmatrix} 2 & -3 & 5 \\ 0 & 1 & -3 \\ 0 & 0 & 2 \end{bmatrix}$$

2. Solve the following system by using Cramer's rule:

$$\begin{aligned} 4x + 5y &= 2 \\ 11x + y + 2z &= 3 \\ x + 5y + 2z &= 1 \end{aligned}$$

3. Prove that if  $\det(A) = 1$  and all entries in  $A$  are integers, then all the entries in  $A^{-1}$  are also integers.

4. Evaluate the following determinants by reducing to row echelon form:

$$\begin{vmatrix} 0 & 3 & 1 \\ 1 & 1 & 2 \\ 3 & 2 & 4 \end{vmatrix}, \quad \begin{vmatrix} 2 & 1 & 3 & 1 \\ 1 & 0 & 1 & 1 \\ 0 & 2 & 1 & 0 \\ 0 & 1 & 2 & 3 \end{vmatrix}$$

5. Use row reduction to show that

$$\begin{vmatrix} 1 & 1 & 1 \\ a & b & c \\ a^2 & b^2 & c^2 \end{vmatrix} = (b-a)(c-a)(c-b)$$

6. Let  $A$  be an  $n \times n$  invertible matrix. Find the determinant of  $2(A^2)^{-1}$  in terms of  $\det(A)$ .

7. Using determinants, find the equation of the line through the points  $(1, 3)$  and  $(2, 5)$ .

8. Find the equation of the conic section passing through  $(0, 0)$ ,  $(0, -1)$ ,  $(2, 0)$ ,  $(2, -5)$  and  $(4, -1)$ . What type of conic is it?

9. Find the equation of the plane passing through  $(2, 3, 1)$ ,  $(2, -1, -1)$ ,  $(1, 2, 1)$ .