M E T U
Northern Cyprus Campus

	Calculus for Functions of Several Variables Short Exam 2	

Code : *Math 120* Acad.Year: *2012-2013* Semester : *Summer* Date : *25.07.2013* Time : *18:45* Duration : *40 minutes*	Last Name: Name: Student No: Signature: 5 QUESTIONS ON 2 PAGES TOTAL 20 POINTS

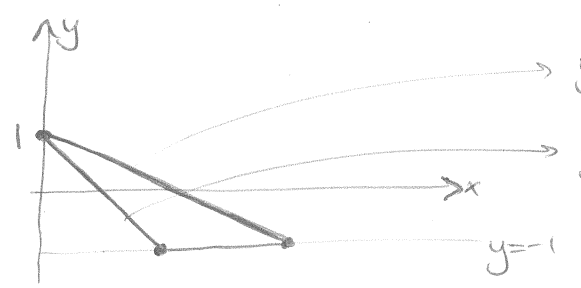
1	2	3	4	5			
					KEY		

Show your work! No calculators! Please draw a $\boxed{\text{box}}$ around your answers!
Please do not write on your desk!

1. ($2\times 2 = 4$ pts.) Let T be the triangle in the cartesian 2−space with vertices $(0,1)$, $(2,-1)$, and $(4,-1)$, and suppose $f(x,y)$ is a continuous function on T. Find $\alpha, \beta, \gamma, \theta$ so that $\iint_T f(x,y)\,dA = \int_\alpha^\beta \int_\gamma^\theta f(x,y)\,dx\,dy$

$\alpha =$ -1 ; $\beta =$ $+1$; $\gamma =$ $1-y$; $\theta =$ $2-2y$

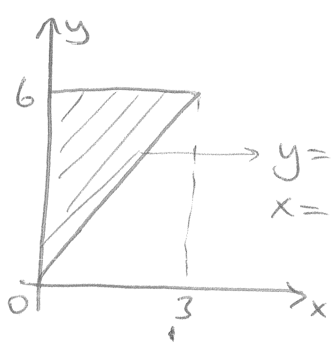
DO NOT EVALUATE THIS INTEGRAL.



$y = \frac{-x}{2} + 1 \quad \longleftrightarrow \quad x = 2-2y$

$y = -x+1 \quad \longleftrightarrow \quad x = 1-y$

2. (4 pts.) Evaluate the integral $\int_0^3 \int_{2x}^6 e^{4x/y}\,dy\,dx$ by reversing the order of integration.



$y = 2x$
$x = y/2$

$$I = \int_{y=0}^{y=6} \int_{x=0}^{x=y/2} e^{4x/y}\,dx\,dy$$

$$= \int_0^6 \left(\frac{e^{4x/y}}{4/y} \right) \Bigg|_{x=0}^{x=y/2} dy$$

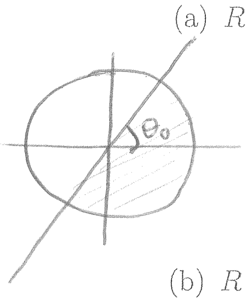
$$= \int_0^6 \frac{y}{4}\left(e^2 - e^0\right) dy$$

$$= \frac{1}{4}\left(e^2-1\right) \cdot \frac{y^2}{2} \Bigg|_0^6$$

$$= \frac{9}{2}\left(e^2-1\right)$$

3. ($2\times 2 = 4$ pts.) Convert the integral $\iint_R f(x,y)\,dA$ to an integral in **polar coordinates**.

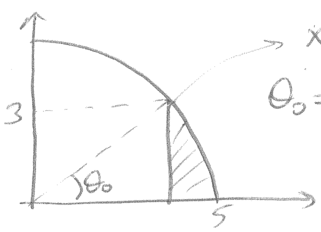
(a) R is the region inside the circle $x^2+y^2=1$ that is below the line $y=\sqrt{3}x$.



$\theta_0 = \arctan(\sqrt{3}) = \pi/3$

$$\int_{\theta=-2\pi/3}^{\theta=\pi/3} \int_{r=0}^{r=1} f(r\cos\theta, r\sin\theta)\cdot r\,dr\,d\theta$$

(b) R is the region in the first quadrant inside the circle $x^2+y^2=25$ that is to the right of the line $x=4$.



$x=4 \Leftrightarrow r = 4/\cos\theta$

$\theta_0 = \arctan(3/4)$

$$\int_{\theta=0}^{\theta=\arctan(3/4)} \int_{r=4/\cos\theta}^{r=5} f(r\cos\theta, r\sin\theta)\,r\,dr\,d\theta$$

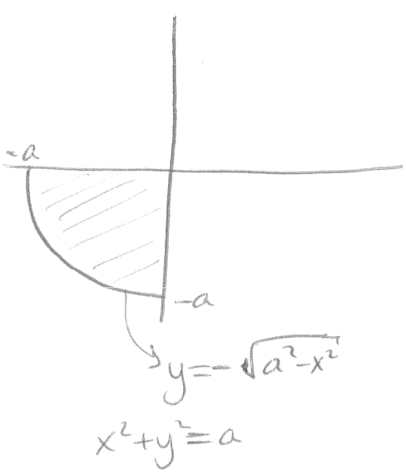
4. (4 pts.) Use the change of variables $s=2x+3y$, $t=2y$ to find the area of the ellipse $4x^2+12xy+13y^2\le 3$. $\rightarrow E$

$$\frac{\partial(s,t)}{\partial(x,y)} = \begin{vmatrix} 2 & 3 \\ 0 & 2 \end{vmatrix} = 4 \qquad \underbrace{s^2+t^2\le 3}_{D}$$

$$\text{Area of } E = \iint_E 1\cdot dA = \iint_E 1\cdot dx\,dy = \iint_D 1\cdot\left|\frac{1}{4}\right|\cdot ds\,dt$$

$$= \frac{1}{4}\text{ Area of } D = \frac{1}{4}\cdot\pi\cdot(\sqrt{3})^2 = \frac{3\pi}{4}$$

5. (4 pts.) Use **polar coordinates** to evaluate $\underbrace{\int_{-a}^{0}\int_{-\sqrt{a^2-x^2}}^{0} e^{x^2+y^2}\,dy\,dx}_{I}$ $(a>0)$



$$I = \int_{\cancel{\pi}}^{3\pi/2}\int_0^a e^{r^2}\,r\,dr\,d\theta \qquad \begin{aligned} u&=r^2\\ du&=2r\,dr\end{aligned}$$

$$= \frac{\pi}{2}\int_{u=0}^{u=a^2} \frac{e^u\,du}{2}$$

$$= \frac{\pi}{4}\; e^{a^2} - e^0 = \frac{\pi(e^{a^2}-1)}{4}$$