10. *(13 pts)* Let $f(x) = \dfrac{2x^2-4}{x^2-4x+3}$.

A) What is the domain of $f(x)$?

$x^2-4x+3=0 \Rightarrow (x-3)(x-1)=0 \Rightarrow x=3;1 \Rightarrow D_f = \mathbb{R}-\{1,3\}$

B) What is the y-intercept of $f(x)$?

$f(0) = -\frac{4}{3} \leftarrow$ y-int

C) What are the x-intercepts of $f(x)$?

$2x^2-4=0 \Rightarrow x=\mp\sqrt{2} \leftarrow$ x-int

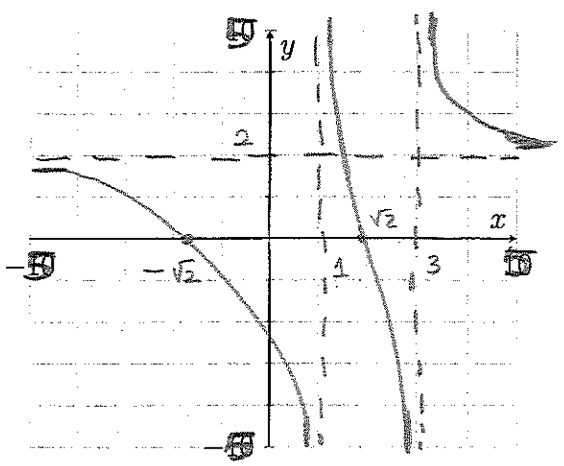
D) For a rational function, such as $f(x)$, the vertical asymptotes are the vertical lines corresponding to the roots of its denominator. Find the vertical asympotes of $f(x)$.

$x^2-4x+3=0 \Rightarrow x=1$ and $x=3$ are vertical asymptotes.

E) For a rational function, such as $f(x)$, has a horizontal asymptote if the numerator and the denominator of the same degree. The horizontal asymptote is the horizontal line which corresponds to the quotient of the numerator divided by the denominator of $f(x)$. Find the horizontal asymptote of $f(x)$.

$\dfrac{2x^2-4}{x^2-4x+3} = 2 + \dfrac{8x-10}{x^2-4x+3} \Rightarrow y=2$ is horizontal asym.

F) Sketch the graph of $f(x)$.



	$-\sqrt{2}$		1	$\sqrt{2}$	3	
$x+\sqrt{2}$	$-$ 0	$+$	$+$	$+$		$+$
$x-\sqrt{2}$	$-$	$-$	$-$ 0	$+$		$+$
$x-1$	$-$	$-$ 0	$+$	$+$		$+$
$x-3$	$-$	$-$	$-$	$-$ 0		$+$
$f(x)$	$+$ 0	$-$	$+$ 0	$-$		$+$

V.A V.A

METU - NCC

	Precalculus Midterm	
Code : *Math 100*	Last Name:	
Acad. Year: *2013-2014*	Name :	Student No.:
Semester : *Fall*	Department:	Section:
Date : *26.11.2012*	Signature:	
Time : *17:40*	10 QUESTIONS ON 4 PAGES	
Duration : *100 minutes*	TOTAL 100 POINTS	

1 (10)	2 (8)	3 (12)	4 (8)	5 (10)	6 (12)	7 (10)	8 (8)	9 (9)	10 (13)			

1. *(10 pts)* Find all x satisfying $|3x+6| \geq x + |x|$.

when $x \leq -2$; $-(3x+6) \geq x + (-x)$

$\Rightarrow x \leq -2$

when $-2 < x \leq 0$; $3x+6 \geq x + (-x)$

$\Rightarrow x \geq -2$

when $x > 0$; $3x+6 \geq x + x$

$\Rightarrow x \geq -6$

$\boxed{\text{Solution: } \mathbb{R}}$

2. *(8 pts)* The sum of squares of two integers equals 212. If their arithmetic mean (average) is 5 then find the bigger one?

$x^2+y^2=212$

$\frac{x+y}{2} = 5 \Rightarrow y = 10-x$

$\Rightarrow x^2 + (10-x)^2 = 212$

$x^2 + x^2 - 20x + 100 = 212$

$2x^2 - 20x - 112 = 0 \Rightarrow x^2 - 10x - 56 = 0$

$\Rightarrow (x-14)(x+4) = 0 \Rightarrow x = 14;\ x = -4$

$y = -4;\ y = 14$

$\boxed{\text{bigger one} = 14}$

3. *(12 pts)* Find all solutions of x satisfying

$$x^2+3x-8 = 2\sqrt{x^2+3x}$$

Let $\sqrt{x^2+3x} = a$; $a^2 - 8 = 2a \Rightarrow a^2 - 2a - 8 = 0$

$(a-4)(a+2) = 0$

$a = 4;\ a = -2$

So, $\sqrt{x^2+3x} = -2$ Not possible!

$\sqrt{x^2+3x} = 4 \Rightarrow x^2+3x-16 = 0$

$x = \dfrac{-3 \mp \sqrt{9+64}}{2}$ — $\dfrac{-3-\sqrt{73}}{2}$, $\dfrac{-3+\sqrt{73}}{2}$

$\boxed{\text{Solutions: } \frac{-3-\sqrt{73}}{2};\ \frac{-3+\sqrt{73}}{2}}$

4. *(8 pts)* Find a nonconstant polynomial $P(x)$ so that $P(1) = P(2) = P(3) = 4$.

$P(x) = (x-1)(x-2)(x-3) + 4$

OR

$P(x) = x^3 - 6x^2 + 11x - 6$

$\boxed{P(x) = (x-1)(x-2)(x-3) + 4}$

5. *(10 pts)* Divide $(x^2+1)(x^2+4x+3)+4$ by $x+1$. What are the quotient and the remainder?

$$\frac{(x^2+1)(x^2+4x+3)+4}{x+1} = \frac{(x^2+1)(x+3)(x+1)+4}{x+1}$$

$$= (x^2+1)(x+3) + \frac{4}{x+1}$$

Quotient $= (x^2+1)(x+3)$

Remainder $= 4$

6. *(12 pts)* The vertices of a square $ABCD$ are $A(a,7)$, $B(2,3)$, $C(6,c)$ and $D(3,10)$. Find a, c and area of the square.

Midpoint of AC = Midpoint of BD

$$\left(\frac{a+6}{2}, \frac{7+c}{2}\right) = \left(\frac{2+3}{2}, \frac{3+10}{2}\right)$$

$\Rightarrow a = -1$; $c = 6$

One side length (like $|AB|$) $= \sqrt{(-1-2)^2 + (7-3)^2} = 5$

So its area is 25.

$a = -1$

$c = 6$

Area $= 25$

7. *(10 pts)* What is the equation of the line which passes through the vertex of the parabola $y = 3x^2 - 12x + 9$ and the point $(0,1)$?

$y = 3(x-2)^2 - 3 \Rightarrow$ vertex : $(2,-3)$

Line eqn: $y = mx + b$, $m = \dfrac{1-(-3)}{0-2} = -2$

$\Rightarrow y = -2x + b$

$1 = -2 \cdot 0 + b \Rightarrow b = 1$

$\boxed{y = -2x + 1}$

Line eqn : $y = -2x + 1$

8. *(8 pts)* Complete the table below by filling in correct values of $(f \circ f)(x)$ and $(g \circ f^{-1})(x)$:

x	1	2	3	4
$f(x)$	2	1	4	3
$g(x)$	2	4	1	4
$(f \circ f)(x)$	1	2	3	4
$(g \circ f^{-1})(x)$	4	2	4	1

9. *(9 pts)* Let $f(x) = x^2 - 4$ for $x \le 0$. The function $f(x)$ is a one-to-one function. What is its inverse, $f^{-1}(x)$? Find domain and range of $f^{-1}(x)$.

$y = x^2 - 4$; $x \le 0$

To find f^{-1}, swap x & y ;

$x = y^2 - 4$; $y \le 0$

$\Rightarrow y^2 = x + 4$; $y \le 0$

$\Rightarrow y = -\sqrt{x+4}$ (since $y \le 0$)

$\Rightarrow f^{-1}(x) = -\sqrt{x+4} \Rightarrow D_{f^{-1}} = [-4, \infty)$

$R_{f^{-1}} = (-\infty, 0]$

$f^{-1}(x) = -\sqrt{x+4}$

Domain $f^{-1}(x) = [-4, \infty)$

Range $f^{-1}(x) = (-\infty, 0]$