M E T U
Northern Cyprus Campus

	Calculus with Analytic Geometry Short Exam 1	
Code : *Math 119* Acad.Year: *2013-2014* Semester : *Summer* Date : *08.07.2013* Time : *18:00* Duration : *30 minutes*	Last Name: Name: Key Student No: Signature: 3 QUESTIONS 2 PAGES TOTAL 20 POINTS	

1(6)	2(8)	3(6)			

Show your work! No calculators! Please draw a $\boxed{\text{box}}$ around your answers!
Please do not write on your desk!

1. ($3 \times 2 = 6$ pts.) Evaluate the limit, if it exists. **Give reasoning**.

(a) $\lim_{x \to 1} \frac{\sqrt{x}-1}{x^2-\sqrt{x}} = \lim_{x\to 1}\left(\frac{\sqrt{x}-1}{x^2-\sqrt{x}} \cdot \frac{\sqrt{x}+1}{\sqrt{x}+1} \cdot \frac{x^2+\sqrt{x}}{x^2+\sqrt{x}}\right)$

$$= \lim_{x\to 1} \frac{(x-1)(x^2+\sqrt{x})}{(x^4-x)(\sqrt{x}+1)} = \lim_{x\to 1} \frac{(x-1)(x^2+\sqrt{x})}{x(x-1)(x^2+x+1)(\sqrt{x}+1)}$$

$$\overset{(x\neq 1)}{=} \lim_{x\to 1} \frac{(x^2+\sqrt{x})}{x(x^2+x+1)(\sqrt{x}+1)} = \frac{2}{3.2} = \boxed{\frac{1}{3}}$$

(b) $\lim_{x \to -1} \frac{x^3-8}{(x+1)(x+2)^2} = \lim_{x\to -1} \frac{(x-2)(x^2+2x+4)}{(x+1)(x+2)^2}$

	-2		-1		2	
+		+		−		+

$$\left.\begin{aligned} \lim_{x\to -1^+} \frac{(x-2)(x^2+2x+4)}{(x+1)(x+2)^2} &= -\infty \\ &\neq \\ \lim_{x\to -1^-} \frac{(x-2)(x^2+2x+4)}{(x+1)(x+2)^2} &= +\infty \end{aligned}\right\} \Rightarrow \lim_{x\to 1} \frac{x^3-8}{(x+1)(x+2)^2} \quad \text{DNE}$$

(c) $\lim_{x \to 0} \frac{|3x-2|-|3x+2|}{x}$

$$|3x-2| = \begin{cases} 3x-2 & \text{if } x \geq 2/3 \\ -3x+2 & \text{if } x < 2/3 \end{cases}, \quad |3x+2| = \begin{cases} 3x+2 & \text{if } x \geq -2/3 \\ -3x-2 & \text{if } x < -2/3 \end{cases}$$

$$\lim_{x\to 0} \frac{|3x-2|-|3x+2|}{x} = \lim_{x\to 0} \frac{-3x+2-(3x+2)}{x}$$

$$= \lim_{x\to 0} \frac{-6x}{x} \underset{(x\neq 0)}{=} \lim_{x\to 0} -6 = \boxed{-6}$$

2. ($8 \times 1 = 8$ pts.) Determine whether the given statement is true or false.
Indicate your answers by typing **TRUE** or **FALSE**. No explanations required.

(a) If $\lim_{x \to a} f(x) = \infty$ and $\lim_{x \to a} g(x) = \infty$, then $\lim_{x \to a}(f(x) - g(x)) = 0$.

FALSE

(b) If $\lim_{x \to a} f(x)g(x)$ exists then $\lim_{x \to a} f(x)g(x) = f(a)g(a)$.

FALSE

(c) If f is continuous at a then it is differentiable at a.

FALSE

(d) If f is differentiable at a then it has a limit at a.

TRUE

(e) If f is continuous at 5 and $f(5) = 2$ and $f(2) = 3$ then $\lim_{x \to 2} f(4x^2 - 11) = 2$.

TRUE

(f) An equation of the tangent line to the curve $y = x^2$ at the point $(1, 1)$ is $y - 1 = 2x(x - 1)$.

FALSE

(g) $\dfrac{|x^2 - 4|}{|x - 2|} = |x + 2|$

FALSE

(h) $\lim_{x \to 0} \dfrac{|x^2 - 4|}{|x - 2|} = \lim_{x \to 0} |x + 2|$

TRUE

3. (a) (4 pts.) Show that $f'(1) = 6$ if $f(x) = 3x^2 + 1$, *using the limit definition of the derivative only.* (Note : Any other methods will not receive *any* credit.)

$$f'(1) = \lim_{h \to 0} \frac{f(1+h) - f(1)}{h} = \lim_{h \to 0} \frac{3(1+h)^2 + 1 - 4}{h}$$

$$= \lim_{h \to 0} \frac{3 + 6h + 3h^2 - 3}{h} = \lim_{h \to 0} \frac{h(6 + 3h)}{h}$$

$$\overset{(h \neq 0)}{=} \lim_{h \to 0} (6 + 3h) = 6$$

(b) (2 pts.) Write the equation of the tangent line to the graph of $f(x) = 3x^2 + 1$ at the point $(1, 4)$.

$y - 4 = 6(x - 1)$, or equivalently $\boxed{y = 6x - 2}$

DID YOU WRITE YOUR NAME AND ID NUMBER ON THE PAPER?