M E T U
Northern Cyprus Campus

	Calculus with Analytic Geometry Short Exam 1	
Code : *Math 119* Acad.Year: *2012-2013* Semester : *Summer* Date : *08.07.2013* Time : *17:45* Duration : *30 minutes*	Last Name: Name: KEY Student No: Signature: 4 QUESTIONS ON 2 PAGES TOTAL 10 POINTS	

1	2	3	4			

Show your work! No calculators! Please draw a $\boxed{\text{box}}$ around your answers!
Please do not write on your desk!

1. (1 pt.) Determine whether the given statement is true or false. If it is true, explain why. If it is false, explain why or give an example that disproves the statement.

If f' is discontinuous, then f cannot be differentiable.

FALSE.

example: $f(x) = \begin{cases} x^2 \sin\frac{1}{x} & x \neq 0 \\ 0 & x = 0 \end{cases}$

2. ($3 \times 1 = 3$ pts.) Evaluate the limit, if it exists. **Give reasoning.**

(a) $\displaystyle\lim_{x\to 3} \frac{x^2-2x-3}{|x-3|}$

$\displaystyle\lim_{x\to 3^+} \frac{(x-3)(x+1)}{x-3} = \lim_{x\to 3^+} x+1 = 4$

$|x-3| = \begin{cases} x-3 & x \geq 3 \\ -x+3 & x < 3 \end{cases}$

$\displaystyle\lim_{x\to 3^-} \frac{(x-3)(x+1)}{-x+3} = \lim_{x\to 3^-} -(x+1) = -4.$

$\displaystyle\Rightarrow \lim_{x\to 3} \frac{x^2-2x-3}{|x-3|}$ $\boxed{\text{dne}}$

(b) $\displaystyle\lim_{x\to -1} \frac{x^3-1}{x^2(x+1)} = \lim_{x\to -1} \frac{(x-1)(x^2+x+1)}{x^2(x+1)}$

-1	0	1	
+	−	−	+

$\displaystyle\lim_{x\to -1^+} \frac{x^3-1}{x^2(x+1)} = -\infty \neq \lim_{x\to -1^-} \frac{x^3-1}{x^2(x+1)} = \infty$

$\displaystyle\Rightarrow \lim_{x\to -1} \frac{x^3-1}{x^2(x+1)}$ $\boxed{\text{d.n.e}}$

(c) $\displaystyle\lim_{x\to 2} \frac{\sin(x^2-2)}{(x^2-2)} = \frac{\sin \lim_{x\to 2}(x^2-2)}{\lim_{x\to 2}(x^2-2)} = \boxed{\frac{\sin 2}{2}}$

($\displaystyle\lim_{x\to 2} x^2-2 = 2$
sin is continuous)

3. (3 pts.) Find the tangent line to the graph of $x^2 + 3xy + y^2 = 5$ at the point $(1, 1)$.

To find $\frac{dy}{dx}$ at $(x, y) = (1, 1)$, use implicit diff.

$2x + 3(y + xy') + 2yy' = 0$

at $(x,y) = (1,1)$ $\quad 2 + 3(1 + y'|_{(1,1)}) + 2y'|_{(1,1)} = 0 \Rightarrow y'|_{(1,1)} = -1$

So tangent line eqn is

$(y - 1) = -1(x - 1)$

$\Rightarrow \boxed{y = -x + 2}$

4. (3 pts.) Find the numbers at which $f(x)$ is discontinuous. **Give reasoning.**

$$f(x) = \begin{cases} \cos(x) & \text{if } x < -1 \\ -1/x & \text{if } -1 \le x < 1 \\ x^3 - 2 & \text{if } 1 \le x < 2 \\ x + 4 & \text{if } x > 2 \end{cases}$$

$\cos(x)$, $x^3 - 2$, $x + 4$ is continuous everywhere.

$-\frac{1}{x}$ is not continuous at $0 \in [-1, 1)$.

$\lim_{x \to -1^-} f(x) = \lim_{x \to -1^-} \cos x = \cos(-1) = \cos(1)$.

$\neq$

$\lim_{x \to -1^+} f(x) = \lim_{x \to -1^+} -\frac{1}{x} = 1 = f(-1)$

So f is discont at $x = -1$.

$\left.\begin{array}{l} \lim_{x \to +1^+} f(x) = \lim_{x \to 1^+} x^3 - 2 = -1 \\ \lim_{x \to +1^-} f(x) = \lim_{x \to 1^-} -\frac{1}{x} = -1 \end{array}\right\} \Rightarrow \lim_{x \to 1} f(x) = -1$

$f(1) = -1 = \lim_{x \to 1} f(x)$ so f is cont at $x = 1$.

f is not defined at 2, so it is discont at 2.

So f is discont at $\boxed{-1, 0, 2}$