

METU - NCC

Precalculus Midterm	
Code : <i>Math 100</i>	Last Name:
Acad. Year: <i>2012-2013</i>	Name : Student No.:
Semester : <i>Spring</i>	Department: Section:
Date : <i>21.04.2013</i>	Signature:
Time : <i>09:40</i>	10 QUESTIONS ON 4 PAGES TOTAL 100 POINTS
Duration : <i>100 minutes</i>	
1 (9) 2 (9) 3 (9) 4 (9) 5 (9) 6 (9) 7 (9) 8 (10) 9 (10) 10 (12) 11 (5)	

1. (9 pts) Find the equation of the line passing through (2,-3) and perpendicular to the line $y = 2x + 5$.

$$y - (-3) = m(x - 2)$$

$$m = -\frac{1}{\text{slope of } y = 2x + 5} = -\frac{1}{2}$$

$$y = -\frac{1}{2}x - 2$$

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2. (9 pts) Find all x satisfying $|x - 3| - |x - 1| = 2$

$$x \geq 3 \quad (x-3) - (x-1) = 2 \Rightarrow -2 = 2 \quad \text{No solution.}$$

$$1 \leq x < 3 \quad -(x-3) - (x-1) = 2 \Rightarrow -2x + 4 = 2 \Rightarrow x = 1$$

$$x < 1 \quad -(x-3) - (-(x-1)) = 2 \Rightarrow 2 = 2 \quad \text{All the numbers less than 1}$$

$$(-\infty, 1]$$

3. (9 pts) Write the complex number $(i^{2012} - i^{2013})(1 + i) + (2 - 4i)\frac{4}{-2 - i}$ in the standard form, i.e. $a + bi$.

$$i^{2012} = (i^4)^{503} = 1 = 1 \quad i^{2013} = (i^4)^{503} \cdot i = i$$

So, we get

$$(1-i)(1+i) + 2(1-2i) \cdot \frac{4(2-i)}{-(2+i)(2-i)}$$

$$2 + 8i$$

$$(1+1) + 2 \cdot \frac{(1-2i)(2-i) \cdot 4}{-5}$$

$$= 2 + 2 \frac{(2 - 2 - 4i - i) \cdot 4}{-5} = 2 - 2(-4i)$$

4. (9 pts) Find three consecutive even integers so that the first plus twice the second is twice the third.

$$n, n+2, n+4$$

$$n+2(n+2) = 2 \cdot (n+4)$$

$$3n+4 = 2n+8$$

$$n = 4$$

4, 6, 8

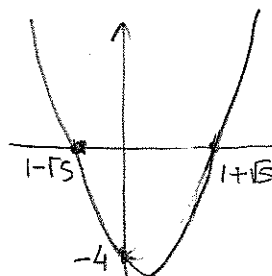
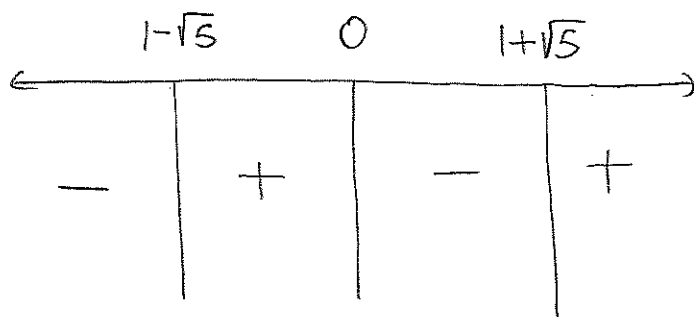
5. (9 pts) Find the set of solutions to the inequality $\frac{x^2 - 2x - 4}{x^3 + 3x} \leq 0$

$$x^2 - 2x - 4 = 0$$

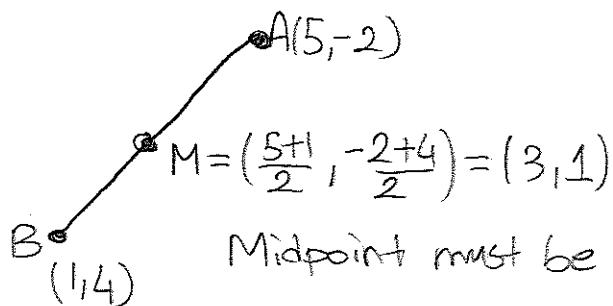
$$\Delta = (-2)^2 - 4(-4) \cdot 1 = 20 \quad x_{1,2} = \frac{2 \pm \sqrt{20}}{2} = 1 \pm \sqrt{5}$$

$$x^3 + 3x = x(x^2 + 3) = 0 \Rightarrow x = 0$$

$(-\infty, 1 - \sqrt{5}] \cup (0, 1 + \sqrt{5}]$



6. (9 pts) Let the points $A=(5, -2)$ and $B=(1, 4)$ on the circle C . If $|AB|$ is equal to the diameter(ap), then find the standard equation of the circle C .



$(x-3)^2 + (y-1)^2 = 13$

Midpoint must be the center.

$$\begin{aligned} \text{radius} = d(M, A) = d(M, B) &= \sqrt{(3-5)^2 + (1-(-2))^2} \\ &= \sqrt{4+9} = \sqrt{13} \end{aligned}$$

7. (9 pts) Divide $P(x) = 3x^4 + 2x^2 - 4$ by $D(x) = x^2 + x$ and find the quotient and the remainder.

$$\begin{array}{r}
 3x^4 + 2x^2 - 4 \quad | \quad x^2 + x \\
 \underline{3x^4 + 3x^3} \\
 -3x^3 + 2x^2 - 4 \\
 \underline{-3x^3 - 3x^2} \\
 5x^2 - 4 \\
 \underline{-5x^2 + 5x} \\
 -5x - 4
 \end{array}$$

$$Q(x) = 3x^2 - 3x + 5$$

$$R(x) = -5x - 4$$

8. (2x5=10 pts) Let $f(x) = 3x + 2$, $g(x) = \frac{x+3}{x}$ and $h(x) = \sqrt{4+x}$. Compute the following values, if defined.

$(f+g)(1)$	9
$(f \circ g)(3)$	8
$(g \circ g)(-3)$	Not Defined
$(g \circ h \circ f)(1)$	2
$(g \circ (\frac{f}{h}))(0)$	4

9. (5+5=10 pts) Given $f(x) = 2x^2 + 4x - 1$
 (a) Find a, h, k so that $f(x) = a(x+h)^2 + k$

$$\begin{aligned}
 2x^2 + 4x - 1 &= 2(x^2 + 2x + 1 - 1) - 1 \\
 &= 2(x+1)^2 - 3
 \end{aligned}$$

$$a = 2 \quad h = 1 \quad k = -3$$

- (b) Use (a) to compute $f^{-1}(x)$

$$1) \quad y = 2x^2 + 4x - 1 = 2(x+1)^2 - 3$$

$$2) \quad x = 2(y+1)^2 - 3$$

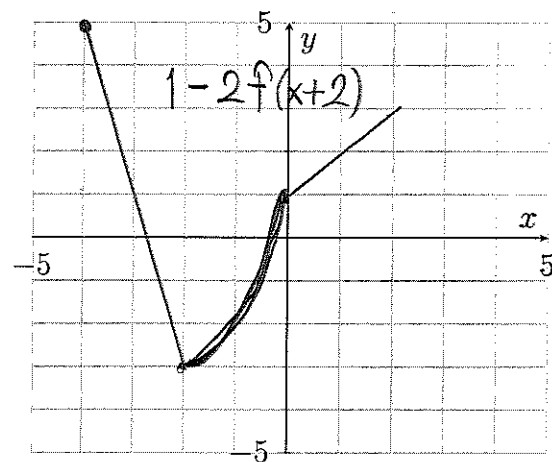
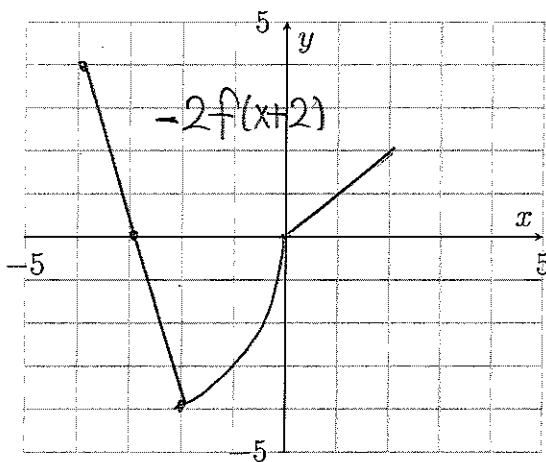
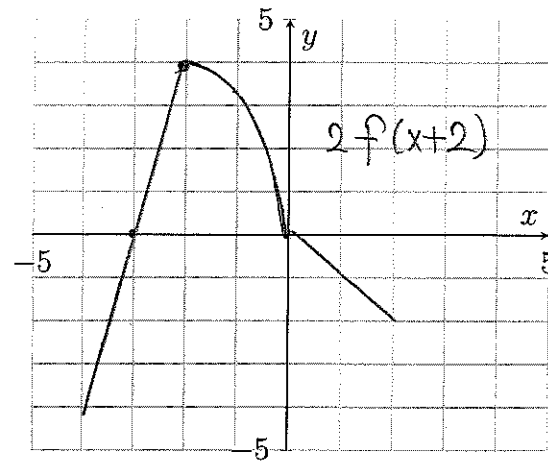
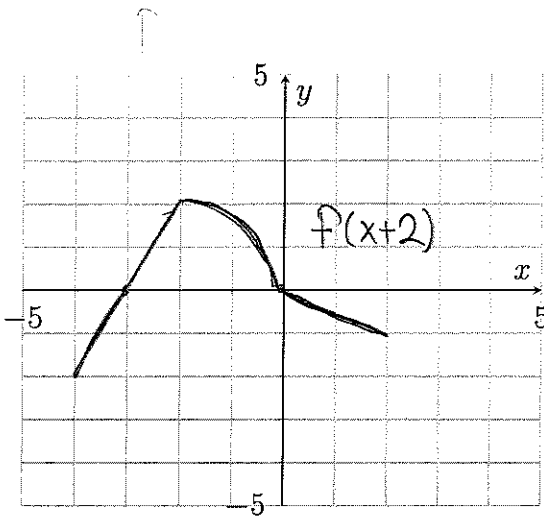
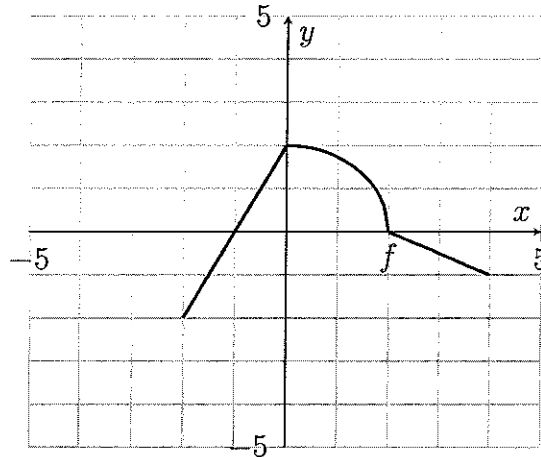
$$3) \quad x+3 = 2(y+1)^2$$

$$\frac{x+3}{2} = (y+1)^2 \quad y = \pm \sqrt{\frac{x+3}{2}} - 1$$

$$f^{-1}(x) = \pm \sqrt{\frac{x+3}{2}} - 1$$

(Both are accepted.)

10. (12 pts) The graph of $f(x)$ is given below. Graph $g(x) = 1 - 2f(x+2)$ in the blank grid. Show individual steps of the transformation of the graph of $f(x)$ to the graph of $g(x)$.



11. (5 pts) Describe a function $g(x)$ in terms of $f(x)$ if the graph of g is obtained by reflecting the graph of f with respect to $y = x$ line first and then shifting 3 units down.