

## 12.3 Exercises

1. Which of the following expressions are meaningful? Which are meaningless? Explain.

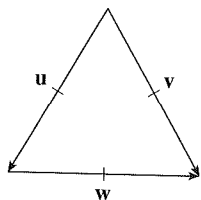
- (a)  $(\mathbf{a} \cdot \mathbf{b}) \cdot \mathbf{c}$  (b)  $(\mathbf{a} \cdot \mathbf{b})\mathbf{c}$   
 (c)  $|\mathbf{a}|(\mathbf{b} \cdot \mathbf{c})$  (d)  $\mathbf{a} \cdot (\mathbf{b} + \mathbf{c})$   
 (e)  $\mathbf{a} \cdot \mathbf{b} + \mathbf{c}$  (f)  $|\mathbf{a}| \cdot (\mathbf{b} + \mathbf{c})$

2–10 Find  $\mathbf{a} \cdot \mathbf{b}$ .

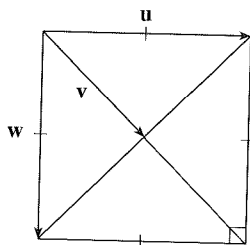
2.  $\mathbf{a} = \langle -2, 3 \rangle$ ,  $\mathbf{b} = \langle 0.7, 1.2 \rangle$   
 3.  $\mathbf{a} = \langle -2, \frac{1}{3} \rangle$ ,  $\mathbf{b} = \langle -5, 12 \rangle$   
 4.  $\mathbf{a} = \langle 6, -2, 3 \rangle$ ,  $\mathbf{b} = \langle 2, 5, -1 \rangle$   
 5.  $\mathbf{a} = \langle 4, 1, \frac{1}{4} \rangle$ ,  $\mathbf{b} = \langle 6, -3, -8 \rangle$   
 6.  $\mathbf{a} = \langle s, 2s, 3s \rangle$ ,  $\mathbf{b} = \langle t, -t, 5t \rangle$   
 7.  $\mathbf{a} = \mathbf{i} - 2\mathbf{j} + 3\mathbf{k}$ ,  $\mathbf{b} = 5\mathbf{i} + 9\mathbf{k}$   
 8.  $\mathbf{a} = 3\mathbf{i} + 2\mathbf{j} - \mathbf{k}$ ,  $\mathbf{b} = 4\mathbf{i} + 5\mathbf{k}$   
 9.  $|\mathbf{a}| = 6$ ,  $|\mathbf{b}| = 5$ , the angle between  $\mathbf{a}$  and  $\mathbf{b}$  is  $2\pi/3$   
 10.  $|\mathbf{a}| = 3$ ,  $|\mathbf{b}| = \sqrt{6}$ , the angle between  $\mathbf{a}$  and  $\mathbf{b}$  is  $45^\circ$

11–12 If  $\mathbf{u}$  is a unit vector, find  $\mathbf{u} \cdot \mathbf{v}$  and  $\mathbf{u} \cdot \mathbf{w}$ .

11.



12.



13. (a) Show that  $\mathbf{i} \cdot \mathbf{j} = \mathbf{j} \cdot \mathbf{k} = \mathbf{k} \cdot \mathbf{i} = 0$ .  
 (b) Show that  $\mathbf{i} \cdot \mathbf{i} = \mathbf{j} \cdot \mathbf{j} = \mathbf{k} \cdot \mathbf{k} = 1$ .  
 14. A street vendor sells  $a$  hamburgers,  $b$  hot dogs, and  $c$  soft drinks on a given day. He charges \$2 for a hamburger, \$1.50 for a hot dog, and \$1 for a soft drink. If  $\mathbf{A} = \langle a, b, c \rangle$  and  $\mathbf{P} = \langle 2, 1.5, 1 \rangle$ , what is the meaning of the dot product  $\mathbf{A} \cdot \mathbf{P}$ ?

15–20 Find the angle between the vectors. (First find an exact expression and then approximate to the nearest degree.)

15.  $\mathbf{a} = \langle 4, 3 \rangle$ ,  $\mathbf{b} = \langle 2, -1 \rangle$   
 16.  $\mathbf{a} = \langle -2, 5 \rangle$ ,  $\mathbf{b} = \langle 5, 12 \rangle$   
 17.  $\mathbf{a} = \langle 3, -1, 5 \rangle$ ,  $\mathbf{b} = \langle -2, 4, 3 \rangle$   
 18.  $\mathbf{a} = \langle 4, 0, 2 \rangle$ ,  $\mathbf{b} = \langle 2, -1, 0 \rangle$   
 19.  $\mathbf{a} = 4\mathbf{i} - 3\mathbf{j} + \mathbf{k}$ ,  $\mathbf{b} = 2\mathbf{i} - \mathbf{k}$   
 20.  $\mathbf{a} = \mathbf{i} + 2\mathbf{j} - 2\mathbf{k}$ ,  $\mathbf{b} = 4\mathbf{i} - 3\mathbf{k}$

21–22 Find, correct to the nearest degree, the three angles of the triangle with the given vertices.

21.  $P(2, 0)$ ,  $Q(0, 3)$ ,  $R(3, 4)$   
 22.  $A(1, 0, -1)$ ,  $B(3, -2, 0)$ ,  $C(1, 3, 3)$

23–24 Determine whether the given vectors are orthogonal, parallel, or neither.

23. (a)  $\mathbf{a} = \langle -5, 3, 7 \rangle$ ,  $\mathbf{b} = \langle 6, -8, 2 \rangle$   
 (b)  $\mathbf{a} = \langle 4, 6 \rangle$ ,  $\mathbf{b} = \langle -3, 2 \rangle$   
 (c)  $\mathbf{a} = -\mathbf{i} + 2\mathbf{j} + 5\mathbf{k}$ ,  $\mathbf{b} = 3\mathbf{i} + 4\mathbf{j} - \mathbf{k}$   
 (d)  $\mathbf{a} = 2\mathbf{i} + 6\mathbf{j} - 4\mathbf{k}$ ,  $\mathbf{b} = -3\mathbf{i} - 9\mathbf{j} + 6\mathbf{k}$   
 24. (a)  $\mathbf{u} = \langle -3, 9, 6 \rangle$ ,  $\mathbf{v} = \langle 4, -12, -8 \rangle$   
 (b)  $\mathbf{u} = \mathbf{i} - \mathbf{j} + 2\mathbf{k}$ ,  $\mathbf{v} = 2\mathbf{i} - \mathbf{j} + \mathbf{k}$   
 (c)  $\mathbf{u} = \langle a, b, c \rangle$ ,  $\mathbf{v} = \langle -b, a, 0 \rangle$

25. Use vectors to decide whether the triangle with vertices  $P(1, -3, -2)$ ,  $Q(2, 0, -4)$ , and  $R(6, -2, -5)$  is right-angled.

26. Find the values of  $x$  such that the angle between the vectors  $\langle 2, 1, -1 \rangle$  and  $\langle 1, x, 0 \rangle$  is  $45^\circ$ .  
 27. Find a unit vector that is orthogonal to both  $\mathbf{i} + \mathbf{j}$  and  $\mathbf{i} + \mathbf{k}$ .  
 28. Find two unit vectors that make an angle of  $60^\circ$  with  $\mathbf{v} = \langle 3, 4 \rangle$ .

29–30 Find the acute angle between the lines.

29.  $2x - y = 3$ ,  $3x + y = 7$   
 30.  $x + 2y = 7$ ,  $5x - y = 2$

31–32 Find the acute angles between the curves at their points of intersection. (The angle between two curves is the angle between their tangent lines at the point of intersection.)

31.  $y = x^2$ ,  $y = x^3$   
 32.  $y = \sin x$ ,  $y = \cos x$ ,  $0 \leq x \leq \pi/2$

33–37 Find the direction cosines and direction angles of the vector. (Give the direction angles correct to the nearest degree.)

33.  $\langle 2, 1, 2 \rangle$  34.  $\langle 6, 3, -2 \rangle$   
 35.  $\mathbf{i} - 2\mathbf{j} - 3\mathbf{k}$  36.  $\frac{1}{2}\mathbf{i} + \mathbf{j} + \mathbf{k}$   
 37.  $\langle c, c, c \rangle$ , where  $c > 0$

38. If a vector has direction angles  $\alpha = \pi/4$  and  $\beta = \pi/3$ , find the third direction angle  $\gamma$ .

39–44 Find the scalar and vector projections of  $\mathbf{b}$  onto  $\mathbf{a}$ .

39.  $\mathbf{a} = \langle -5, 12 \rangle$ ,  $\mathbf{b} = \langle 4, 6 \rangle$

40.  $\mathbf{a} = \langle 1, 4 \rangle$ ,  $\mathbf{b} = \langle 2, 3 \rangle$

41.  $\mathbf{a} = \langle 3, 6, -2 \rangle$ ,  $\mathbf{b} = \langle 1, 2, 3 \rangle$

42.  $\mathbf{a} = \langle -2, 3, -6 \rangle$ ,  $\mathbf{b} = \langle 5, -1, 4 \rangle$

43.  $\mathbf{a} = 2\mathbf{i} - \mathbf{j} + 4\mathbf{k}$ ,  $\mathbf{b} = \mathbf{j} + \frac{1}{2}\mathbf{k}$

44.  $\mathbf{a} = \mathbf{i} + \mathbf{j} + \mathbf{k}$ ,  $\mathbf{b} = \mathbf{i} - \mathbf{j} + \mathbf{k}$

45. Show that the vector  $\text{orth}_{\mathbf{a}} \mathbf{b} = \mathbf{b} - \text{proj}_{\mathbf{a}} \mathbf{b}$  is orthogonal to  $\mathbf{a}$ . (It is called an **orthogonal projection** of  $\mathbf{b}$ .)

46. For the vectors in Exercise 40, find  $\text{orth}_{\mathbf{a}} \mathbf{b}$  and illustrate by drawing the vectors  $\mathbf{a}$ ,  $\mathbf{b}$ ,  $\text{proj}_{\mathbf{a}} \mathbf{b}$ , and  $\text{orth}_{\mathbf{a}} \mathbf{b}$ .

47. If  $\mathbf{a} = \langle 3, 0, -1 \rangle$ , find a vector  $\mathbf{b}$  such that  $\text{comp}_{\mathbf{a}} \mathbf{b} = 2$ .

48. Suppose that  $\mathbf{a}$  and  $\mathbf{b}$  are nonzero vectors.

- (a) Under what circumstances is  $\text{comp}_{\mathbf{a}} \mathbf{b} = \text{comp}_{\mathbf{b}} \mathbf{a}$ ?  
 (b) Under what circumstances is  $\text{proj}_{\mathbf{a}} \mathbf{b} = \text{proj}_{\mathbf{b}} \mathbf{a}$ ?

49. Find the work done by a force  $\mathbf{F} = 8\mathbf{i} - 6\mathbf{j} + 9\mathbf{k}$  that moves an object from the point  $(0, 10, 8)$  to the point  $(6, 12, 20)$  along a straight line. The distance is measured in meters and the force in newtons.

50. A tow truck drags a stalled car along a road. The chain makes an angle of  $30^\circ$  with the road and the tension in the chain is 1500 N. How much work is done by the truck in pulling the car 1 km?

51. A woman exerts a horizontal force of 140 N on a crate as she pushes it up a ramp that is 4 m long and inclined at an angle of  $20^\circ$  above the horizontal. Find the work done on the box.

52. Find the work done by a force of 100 N acting in the direction  $N50^\circ W$  in moving an object 5 m due west.

53. Use a scalar projection to show that the distance from a point  $P_1(x_1, y_1)$  to the line  $ax + by + c = 0$  is

$$\frac{|ax_1 + by_1 + c|}{\sqrt{a^2 + b^2}}$$

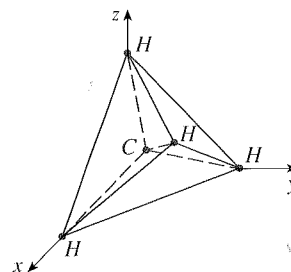
Use this formula to find the distance from the point  $(-2, 3)$  to the line  $3x - 4y + 5 = 0$ .

54. If  $\mathbf{r} = \langle x, y, z \rangle$ ,  $\mathbf{a} = \langle a_1, a_2, a_3 \rangle$ , and  $\mathbf{b} = \langle b_1, b_2, b_3 \rangle$ , show that the vector equation  $(\mathbf{r} - \mathbf{a}) \cdot (\mathbf{r} - \mathbf{b}) = 0$  represents a sphere, and find its center and radius.

55. Find the angle between a diagonal of a cube and one of its edges.

56. Find the angle between a diagonal of a cube and a diagonal of one of its faces.

57. A molecule of methane,  $\text{CH}_4$ , is structured with the four hydrogen atoms at the vertices of a regular tetrahedron and the carbon atom at the centroid. The *bond angle* is the angle formed by the H—C—H combination; it is the angle between the lines that join the carbon atom to two of the hydrogen atoms. Show that the bond angle is about  $109.5^\circ$ . [Hint: Take the vertices of the tetrahedron to be the points  $(1, 0, 0)$ ,  $(0, 1, 0)$ ,  $(0, 0, 1)$ , and  $(1, 1, 1)$ , as shown in the figure. Then the centroid is  $(\frac{1}{2}, \frac{1}{2}, \frac{1}{2})$ .]



58. If  $\mathbf{c} = |\mathbf{a}|\mathbf{b} + |\mathbf{b}|\mathbf{a}$ , where  $\mathbf{a}$ ,  $\mathbf{b}$ , and  $\mathbf{c}$  are all nonzero vectors, show that  $\mathbf{c}$  bisects the angle between  $\mathbf{a}$  and  $\mathbf{b}$ .

59. Prove Properties 2, 4, and 5 of the dot product (Theorem 2).

60. Suppose that all sides of a quadrilateral are equal in length and opposite sides are parallel. Use vector methods to show that the diagonals are perpendicular.

61. Use Theorem 3 to prove the Cauchy-Schwarz Inequality:

$$|\mathbf{a} \cdot \mathbf{b}| \leq |\mathbf{a}| |\mathbf{b}|$$

62. The Triangle Inequality for vectors is

$$|\mathbf{a} + \mathbf{b}| \leq |\mathbf{a}| + |\mathbf{b}|$$

- (a) Give a geometric interpretation of the Triangle Inequality.  
 (b) Use the Cauchy-Schwarz Inequality from Exercise 61 to prove the Triangle Inequality. [Hint: Use the fact that  $|\mathbf{a} + \mathbf{b}|^2 = (\mathbf{a} + \mathbf{b}) \cdot (\mathbf{a} + \mathbf{b})$  and use Property 3 of the dot product.]

63. The Parallelogram Law states that

$$|\mathbf{a} + \mathbf{b}|^2 + |\mathbf{a} - \mathbf{b}|^2 = 2|\mathbf{a}|^2 + 2|\mathbf{b}|^2$$

- (a) Give a geometric interpretation of the Parallelogram Law.  
 (b) Prove the Parallelogram Law. (See the hint in Exercise 62.)

64. Show that if  $\mathbf{u} + \mathbf{v}$  and  $\mathbf{u} - \mathbf{v}$  are orthogonal, then the vectors  $\mathbf{u}$  and  $\mathbf{v}$  must have the same length.