M E T U
Northern Cyprus Campus

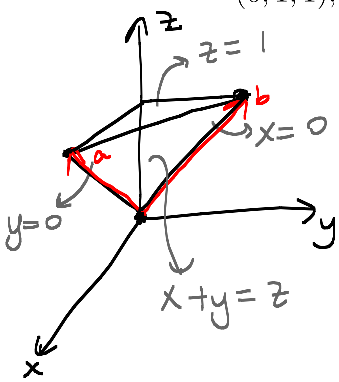
	Calculus for Functions of Several Variables Short Exam 3	
Code : *Math 120* Acad.Year : *2011-2012* Semester : *Spring* Date : *08.8.2012* Time : *19:45* Duration : *45 minutes*	Last Name: Department: Section: Recitation:	Name: Student No: Signature:
	5 QUESTIONS ON 4 PAGES TOTAL 45+2 POINTS	

1	2	3	4	5			

Show your work! No calculators! Please draw a $\boxed{\text{box}}$ around your answers!

Please do not write on your desk!

1. (4 + 4 + 2 = 10 pts.) Consider the tetrahedron T with corner points $(0,0,0)$, $(1,0,1)$, $(0,1,1)$, $(0,0,1)$. Let $I = \iiint_T x^2 z \, dV$.



$$\vec{n} = \vec{a} \times \vec{b} = \begin{vmatrix} i & j & k \\ 1 & 0 & 1 \\ 0 & 1 & 1 \end{vmatrix} = (-1, -1, 1)$$

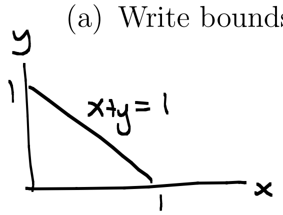
Plane through $(0,0,0)$, $(1,0,1)$, $(0,1,1) = P$

P : $(-1,-1,1) \cdot (x-0, y-0, z-0) = 0$

$-x - y + z = 0$

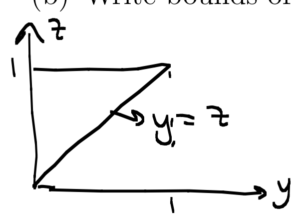
$x + y - z = 0$

(a) Write bounds of the iterated triple integral to calculate I in the $dz\,dy\,dx$ order.



$$\int_{x=0}^{x=1} \int_{y=0}^{y=1-x} \int_{z=x+y}^{z=1} x^2 z \, dz \, dy \, dx$$

(b) Write bounds of the iterated triple integral to calculate I in the $dx\,dz\,dy$ order.



$$\int_{y=0}^{y=1} \int_{z=y}^{z=1} \int_{x=0}^{x=z-y} x^2 z \, dx \, dz \, dy$$

(c) Evaluate any of these integrals. CANCELLED.

$$\frac{1}{72}$$

2. $(2+6+2=10$ pts.) Let $F(x,y) = \Big(\overbrace{2x\sin(xy) + x^2y\cos(xy) + 1}^{P}\,,\ \overbrace{x^3\cos(xy) + 2y}^{Q}\Big)$.

(a) Show that $F(x,y)$ is a conservative vector field.

$$P_y = 2x\cos(xy)\cdot x + x^2\cos(xy) - x^2y\cdot\sin(xy)\cdot x + 0$$

$$Q_x = 3x^2\cos(xy) - x^3y\sin(xy)$$

(b) Find a potential function $f(x,y)$ for $F(x,y)$, i.e., a function that satisfies $\left(\frac{\partial f}{\partial x}, \frac{\partial f}{\partial y}\right) = \Big(2x\sin(xy) + x^2y\cos(xy) + 1\,,\ x^3\cos(xy) + 2y\Big)$.

$$f = \int \left(x^3\cos(xy) + 2y\right) dy = \frac{x^3\sin(xy)}{x} + y^2 + g(x) = x^2\sin(xy) + y^2 + g(x)$$

$$f_x = 2x\cancel{\sin(xy)} + \cancel{x^2\cos(xy)\cdot y} + 0 + g'(x) = 2x\cancel{\sin(xy)} + \cancel{x^2y\cos(xy)} + 1$$

$$g'(x) = 1 \Rightarrow g(x) = x + C$$

$$\Rightarrow \boxed{f = x^2\sin(xy) + y^2 + x + C}$$

(c) Evaluate the line integral $\int_C F(x,y) \bullet dr$ on any curve C that starts from the point $(0,0)$ and that ends at the point $(-1,1)$ **by using Fundamental Theorem of Line Integrals only.** Other methods will not receive any credits.

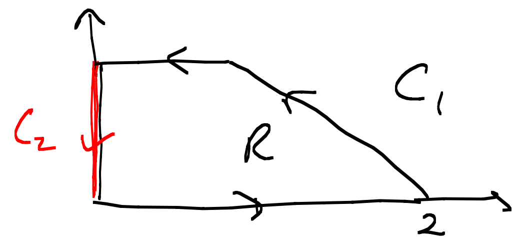
$$\int_C F(x,y)\cdot dr = f(-1,1) - f(0,0)$$

$$= \left((-1)^2\sin(-1) + 1^2 - 1 + C\right) - \left(0 + 0 + 0 + C\right)$$

$$= \sin(-1) = -\sin 1 .$$

3. $(1+2+2+4=9$ pts.) Let R be the region in the first quadrant bounded by the lines $y=1$ and $x+y=2$. Let C_1 be the oriented curve on the boundary of R, consisting of the line segments that go from $(0,0)$ to $(2,0)$ to $(1,1)$ to $(0,1)$. Let C_2 be the line segment from $(0,1)$ to (0.0).

(a) Sketch the configuration on the right.



(b) Find the area of the region R.

$$1+\frac{1}{2}=\frac{3}{2}$$

(c) Evaluate the line integral $\int_{C_2} F(x,y)\bullet dr$, where $F(x,y)=\left(\cos\left(\frac{x^3+1}{x^2+1}\right), 2x+\frac{1}{y^2+1}\right)$.

$$C_2: \quad r(t)=t(0,-1)+(0,1) \quad t\in[0,1], \quad \frac{dr}{dt}=(0,-1)$$

$$\int_{C_2} F\cdot dr = \int_0^1 \left(\cos(\ldots), \frac{1}{t^2+1}\right)\cdot(0,-1)\cdot dt = -\int_0^1 \frac{dt}{t^2+1} = -\arctan t\Big|_0^1$$

$$= \arctan 0 - \arctan 1$$

$$= 0-\frac{\pi}{4} = -\frac{\pi}{4}.$$

(d) Evaluate the line integral $\int_{C_1} F(x,y)\bullet dr$, where $F(x,y)$ is the function in part (b) **by using Green's Theorem only.** Other methods will not receive any credits.

$$\int_{C_1\cup C_2} F\cdot dr = \iint_R (Q_x-P_y)\,dA = \iint_R 2\,dA = 2\cdot \text{Area}(R) = 2\cdot\frac{3}{2}=3.$$

$$\int_{C_1} F\cdot dr + \int_{C_2} F\cdot dr = 3 \Rightarrow \int_{C_1} F\cdot dr = 3-\left(-\frac{\pi}{4}\right) = 3+\frac{\pi}{4}.$$

4. $(8\times 2=16$ pts.) Fill in the blanks according to the rules specified below.
In the first blank space, provide the name of the series test you have used.
If your answer for the test name is "Integral Test", "Comparison Test", or "Limit Comparison Test", then, in the second blank space provide the integral or series you have compared.
In the third blank space, write "C" for convergent or "D" for Divergent.

Example :

Series	Name of the test	Compared with...	Convergent or Divergent
$\sum_{n=1}^{\infty}\frac{1}{n}$	$p-$series	——————-	D

Series	Name of the test	Compared with...	Convergent or Divergent
$\sum_{n=1}^{\infty} n^2$	Test for Divergence	$\lim n^2 = \infty$	D
$\sum_{n=1}^{\infty} \left(\frac{e}{\pi}\right)^{3n-1} = \sum \frac{\pi}{e} \left(\left(\frac{e}{\pi}\right)^3\right)^n$		Geom. Series $e < \pi$	C
$\sum_{n=1}^{\infty} (-1)^n \sqrt{n}$	Test for Divergence	$\lim (-1)^n \sqrt{n}$ DNE	D
$\sum_{n=1}^{\infty} \frac{n}{n^3+4}$	Limit Comparison	$\frac{1}{n^2}$	C
$\sum_{n=1}^{\infty} \frac{\sqrt{n^7+2012}}{\sqrt{n^8+8}}$	Limit Comp	$\frac{1}{\sqrt{n}}$	D
$\sum_{n=2}^{\infty} \frac{1}{\ln(n)}$	Comp	$\ln n < n$ $\frac{1}{\ln} > \frac{1}{n}$	D
$\sum_{n=2}^{\infty} \frac{1}{8^n + n^8 + 2012}$	lim Comp	$\left(\frac{1}{8}\right)^n$ Geom.	C
$\sum_{n=1907}^{\infty} \frac{\cos(n\pi)}{\sqrt{n}} = \sum_{n=1907}^{\infty} \frac{(-1)^n}{\sqrt{n}}$		A.S.T	C

5. (2 pts.) Depending on your performance in this exam, give a closed interval of length five for the grade you are expecting out of the previous 45 points.
You will receive 2 points if your guess is correct.
You will receive 0 points in any other case.
Example :
Arda is expecting a score around 35/45, so his guess is [32,37].
Buğra is expecting a score around 43/45, so his guess is [40,45].

[40,45]