M E T U
Northern Cyprus Campus

	Calculus for Functions of Several Variables Short Exam 3	
Code : *Math 120* Acad.Year: *2012-2013* Semester : *Fall* Date : *03.01.2013* Time : *17:45* Duration : *45 minutes*	Last Name: Name: Student No: Signature: 4 QUESTIONS ON 2 PAGES TOTAL 42+2=44 POINTS	

1	2	3	4	5	KEY		

Show your work! No calculators! Please draw a $\boxed{\text{box}}$ around your answers!
Please do not write on your desk!

1. (8 pts.) Use the Fundamental Theorem of Line Integrals to calculate $\int_C \langle 2xy+1, x^2+2y\rangle \cdot d\vec{r}$ where C is any curve from the point $(1,2)$ to the point $(2,1)$.

Suppose ϕ is the potential function. Then $\phi_x = 2xy+1$, $\phi_y = x^2+2y$

$\phi = \int \phi_x \, dx = \int (2xy+1)\,dx = x^2y + x + g(y) \Rightarrow \phi_y = x^2 + g'(y) = x^2 + 2y$

$\Rightarrow g'(y) = 2y \Rightarrow g(y) = y^2 \Rightarrow \boxed{\phi(x,y) = x^2y + x + y^2}$

Then, $\int_C \langle 2xy+1, x^2+2y\rangle \cdot dr = \phi(2,1) - \phi(1,2)$

$= (2^2\cdot 1 + 2 + 1^2) - (1^2\cdot 2 + 1 + 2^2) = \boxed{0}$

2. ($2 \times 4 = 8$ pts.) Evaluate the following limits.

(a) $\lim_{n\to\infty} (e^{2n} + 6n)^{1/n} = \lim_{x\to\infty} \underbrace{(e^{2x}+6x)^{1/x}}_{y}$

$\lim_{x\to\infty} \ln y = \lim_{x\to\infty} \frac{\ln(e^{2x}+6x)}{x} \overset{(\infty/\infty)}{=} \lim_{x\to\infty} \frac{2e^{2x}+6}{e^{2x}+6x} = \lim_{x\to\infty} \frac{2+\overset{\to 0}{6e^{-2x}}}{1+\underset{\to 0}{6xe^{-2x}}} = 2$

Then $\lim_{x\to\infty} y = \lim_{x\to\infty} e^{\ln y} = e^{\lim \ln y} = \boxed{e^2}$

(b) $\lim_{n\to\infty} \left[\ln(2013n+1) - \ln(3n)\right] = \lim_{n\to\infty} \ln\left(\frac{2013n+1}{3n}\right)$

$= \lim_{n\to\infty} \ln\left(\frac{2013 + 1/n}{3}\right) = \ln \lim_{n\to\infty}\left(\frac{2013+1/n}{3}\right)$

$= \ln\left(\frac{2013}{3}\right) = \boxed{\ln(671)}$

3. ($5 \times 4 = 20$ pts.) Determine if the following series are convergent or divergent. Give brief reasoning. One of these is a bonus question.

(a) $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n^2}$ CONVERGENT $(p=2>1)$ p-series.

(b) $\displaystyle\sum_{n=1}^{\infty} \frac{e^n}{\pi^n}$ $= \displaystyle\sum_{n=1}^{\infty} \left(\frac{e}{\pi}\right)^n$ $e < \pi \Rightarrow \left|\frac{e}{\pi}\right| < 1$

$\Rightarrow$ CONVERGENT (Geometric Series with $r = \frac{e}{\pi}$)

(BONUS) (c) $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^n}{n}$

$$\left.\begin{array}{l} \text{i) } \frac{(-1)^n}{n} \text{ alternating} \\ \text{ii) } \frac{1}{n} \text{ decreasing} \\ \text{iii) } \frac{1}{n} \to 0 \end{array}\right\}$$

CONVERGENT by Alternating Series Test.

(d) $\displaystyle\sum_{n=1}^{\infty} n^2$ DIVERGENT since $\lim_{n\to\infty} n^2 = \infty$ (DNE)

by Test for Divergence.

(e) $\displaystyle\sum_{n=1}^{\infty} (-1)^n \frac{n^2}{n^2+1}$ DIVERGENT since

$\displaystyle\lim_{n\to\infty} \frac{(-1)^n n^2}{n^2+1}$ (DNE)

by Test for Divergence.

4. (8 pts.) Determine if the series $\displaystyle\sum_{n=2}^{\infty} \frac{1}{n\ln(n)}$ is convergent or divergent. **Give reasoning** by quoting which test you have used and show that all hypothesis hold.

Let $f(x) = \dfrac{1}{x\ln(x)}$ $(x \geq 2)$ i) f is continuous

ii) f is decreasing (i.e. check $f'<0$)

iii) $f(x) > 0$ for $x \geq 2$.

By Integral Test, $\displaystyle\int_2^{\infty} \frac{dx}{x\ln(x)}$ and $\displaystyle\sum_{n=2}^{\infty} \frac{1}{n\ln n}$ converge or diverge at the same time

$$\int_2^{\infty} \frac{dx}{x\ln x} = \lim_{R\to\infty} \int_2^{R} \frac{dx}{x\ln x} = \lim_{R\to\infty} \int_{\ln 2}^{\ln R} \frac{du}{u} = \lim_{R\to\infty} \ln u \Big|_{\ln 2}^{\ln R}$$

$$= \lim_{R\to\infty} \left(\ln(\ln R) - \ln(\ln 2)\right) = \infty \text{ (DNE)} \Rightarrow \int_2^{\infty} \frac{dx}{x\ln x} \text{ } \underline{\text{DIV.}}$$

By Integral Test, $\displaystyle\sum_{n=2}^{\infty} \frac{1}{n\ln n}$ is $\boxed{\text{DIVERGENT.}}$