M E T U
Northern Cyprus Campus

	Calculus for Functions of Several Variables Short Exam 4	
Code : *Math 120* Acad.Year: *2011-2012* Semester : *Fall* Date : *03.1.2012* Time : *17:45* Duration : *45 minutes*	Last Name: Name: Department: Student No: Section: Signature: 5 QUESTIONS ON 2 PAGES TOTAL 32 POINTS	

1	2	3	4	5

Show your work! No calculators! Please draw a $\boxed{\text{box}}$ around your answers!

Please do not write on your desk!

1. (2+4+2 = 8 pts.)

(a) Show that the vector field $F(x,y) = \left(2x - \frac{1}{x} + y\right)\mathbf{i} + \left(\frac{1}{y} + x\right)\mathbf{j}$ is conservative.

$\frac{\partial}{\partial y}\left(2x - \frac{1}{x} + y\right) = 1$ & $\frac{\partial}{\partial x}\left(\frac{1}{y} + x\right) = 1 \Rightarrow F$ is conservative

(b) Find a potential function f such that $\nabla f = F$.

$\frac{\partial}{\partial x} f = 2x - \frac{1}{x} + y \Rightarrow f = x^2 - \ln x + xy + h(y)$

$\frac{\partial}{\partial y} f = \frac{1}{y} + x = x + h'(y) \Rightarrow h'(y) = \frac{1}{y} \Rightarrow h(y) = \ln y$

$\Rightarrow f(x,y) = x^2 + \ln y - \ln x + xy = x^2 + \ln(y/x) + xy$

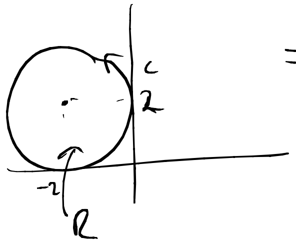
(c) Using parts (a) and (b), evaluate the line integral $\int_C \left(2x - \frac{1}{x} + y\right) dx + \left(\frac{1}{y} + x\right) dy$, where $C : r(t) = \langle t, t^2 \rangle, 1 \le t \le e$.

$I = \int_C F \cdot dr = f(r(e)) - f(r(1)) = f(e, e^2) - f(1,1)$

$= (e^2 + \ln e + e^3) - (1 + \ln 1 + 1) = e^3 + e^2 - 1$

2. (4 pts.) Use Green's Theorem to evaluate $\oint_C \ln(2\pi + \arctan(x))\, dx + \left(2x + \frac{1}{y^2+1}\right) dy$ where C is the circle $(x+2)^2 + (y-2) = 4$ traversed in a counter clockwise fashion.

$I = \oint_C F \cdot dr = \iint_R \left[\frac{\partial}{\partial x}\left(2x + \frac{1}{y^2+1}\right) - \frac{\partial}{\partial y}\left(\ln(2\pi + \arctan x)\right)\right] dA$



$= \iint_R 2\, dA = 2\iint_R dA = 2\,\text{Area}(R)$

$= 2 \cdot \pi 2^2 = 8\pi$

3. ($4 \times 2 = 8$ pts.) Determine whether the given series is convergent or divergent. Give reasoning.

(a) $\sum_{k=1}^{\infty} (\sin 3)^k$

$|\sin 3| < 1 \Rightarrow$ the series is a convergent geometric series.

(b) $\sum_{n=1}^{\infty} \frac{n+5}{\sqrt{n^5 - 2n + 1}}$

Let $b_n = \frac{1}{n^{3/2}}$

$$\lim \frac{\frac{n+5}{\sqrt{n^5-2n+1}}}{1/n^{3/2}} = \lim \frac{n^{5/2}(1+5/n)}{n^{5/2}\sqrt{1-2/n^4+1/n^5}} = \frac{1}{\sqrt{1}} = 1$$

So the original series behaves the same way as $\sum \frac{1}{n^{3/2}}$

But $\sum \frac{1}{n^{3/2}}$ is conv. by p-test. Hence the original series is convergent.

4. (6 pts.) Determine whether $\sum_{n=1}^{\infty} \frac{(-1)^n n}{n^2+1}$ is either absolutely convergent, conditionally convergent, or divergent. Give reasoning.

$* \sum \left| \frac{(-1)^n n}{n^2+1} \right| = \sum \frac{n}{n^2+1}$. $\lim \frac{\frac{n}{n^2+1}}{1/n} = \lim \frac{n^2}{n^2+1} = 1$ & $\sum \frac{1}{n}$ is divergent.

So $\sum \frac{n}{n^2+1}$ is div.

$* \sum \frac{(-1)^n n}{n^2+1}$ AST. 1) $\frac{n}{n^2+1} > 0$ 2) Alt. 3) $\left(\frac{x}{x^2+1}\right)' = \frac{(x^2+1) - x \cdot 2x}{(x^2+1)^4} = \frac{-x^2+1}{(x^2+1)^2} < 0$ for $x > 1$

4) $\frac{n}{n^2+1} \to 0$

By AST the original series is convergent.

So the series is $\boxed{\text{Conditionally Convergent.}}$

5. (6 pts.) Find the radius of convergence and the interval of convergence of $\sum_{n=0}^{\infty} (-1)^n \frac{(x-3)^n}{2n+1}$.

$$\lim \left| \frac{(-1)^{n+1}(x-3)^{n+1}}{2n+3} \cdot \frac{2n+1}{(-1)^n (x-3)^n} \right| = \lim |x-3| \left| \frac{2n+1}{2n+3} \right| = |x-3| < 1 \Rightarrow \text{Radius} = 1$$

Center $= 3$

At $x=2$ $\quad \sum \frac{(-1)^{2n}}{2n+1} = \sum \frac{1}{2n+1}$ is div. (Lim comp. with $\sum 1/n$)

At $x=4$ $\quad \sum \frac{(-1)^n}{2n+1}$ conv. by AST.

$\Rightarrow$ $(2,4]$ is the interval of convergence.

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	Calculus for Functions of Several Variables **Short Exam 4**	
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1. (2+4+2 = 8 pts.)

(a) Show that the vector field $F(x,y) = \left(2xy - \dfrac{1}{x}\right)\mathbf{i} + \left(x^2 + 2ye^{y^2}\right)\mathbf{j}$ is conservative.

$$\frac{\partial}{\partial y}\left(2xy - \frac{1}{x}\right) = 2x = \frac{\partial}{\partial x}\left(x^2 + 2ye^{y^2}\right)$$

(b) Find a potential function f such that $\nabla f = F$.

$$f = \int \frac{\partial}{\partial x} f\, dx = \int \left(2xy - \frac{1}{x}\right) dx = x^2y - \ln x + h(y)$$

$$\frac{\partial f}{\partial y} = 2xy + h'(y) = x^2 + 2ye^{y^2} \Rightarrow h'(y) = 2ye^{y^2} \Rightarrow h(y) = \int 2y\, e^{y^2} dy$$

$$\Rightarrow h(y) = e^{y^2} + C$$

$$\Rightarrow f(x,y) = x^2y - \ln x + e^{y^2} + C$$

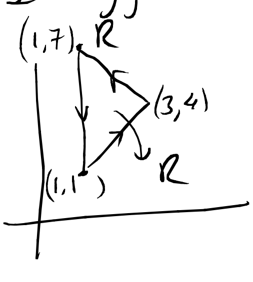
(c) Using parts (a) and (b), evaluate the line integral $\displaystyle\int_C \left(2xy - \frac{1}{x}\right) dx + \left(x^2 + 2ye^{y^2}\right) dy$, where $C : r(t) = \langle t, 2t\rangle, 1 \leq t \leq 2$.

$$\int_C F \cdot dr = f(r(2)) - f(r(1)) = f(2,4) - f(1,2)$$

$$= (4 \cdot 4 - \ln 2 + e^{16} + c) - (1^2 \cdot 2 - \ln 1 + e^4 + c) = 14 + e^{16} - e^4 - \ln 2.$$

2. (4 pts.) Use Green's Theorem to evaluate $\displaystyle\oint_C \left(e^{\pi x^2} + 2y\right) dx + \left(\arctan\left(\sqrt{49 - y^2}\right)\right) dy$ $= I$ where C is composed of the straight line segments from $(1,1)$ to $(3,4)$ to $(1,7)$ traversed in a counter-clockwise fashion.

$$I = \iint (0 - 2)\, dA = -2 \iint_R dA = -2 \cdot \text{Area of } R$$



$$= -2 \cdot \frac{6 \cdot 2}{2} = -12$$

3. ($4 \times 2 = 8$ pts.) Determine whether the given series is convergent or divergent. Give reasoning.

(a) $\sum_{k=1}^{\infty} (\frac{e^2}{\pi^2})^k$ $\quad 0 < e < \pi \Rightarrow e^2 < \pi^2 \Rightarrow \left|\frac{e^2}{\pi^2}\right| = \frac{e^2}{\pi^2} < 1$

$\Rightarrow \sum_{k=1}^{\infty} \left(\frac{e^2}{\pi^2}\right)^k$ is a convergent geometric series.

(b) $\sum_{n=1}^{\infty} \frac{\sqrt{n+5}}{2n^5 - 2n + 1}$ $\quad \frac{\sqrt{n+5}}{2n^5-2n+1} > 0$ lim. comp with $\frac{1}{n^{9/2}}$

$$\lim \frac{\frac{\sqrt{n+5}}{2n^5-2n+1}}{n^{9/2}} = \lim \frac{\sqrt{n^{10}+5n^9}}{2n^5-2n+1} = \lim \frac{n^5\sqrt{1+5/n}}{n^5(2-2/n^4+1/n^5)} = \frac{\sqrt{1}}{2} = \frac{1}{2}$$

$0 < \frac{1}{2} < \infty$ & $\sum_{n=1}^{\infty} \frac{1}{n^{9/2}}$ is conv. by p-test $\Rightarrow$ Original series is convergent

4. (6 pts.) Determine whether $\sum_{n=1}^{\infty}(-1)^n \frac{n}{n^3+1}$ is either absolutely convergent, conditionally convergent, or divergent. Give reasoning.

$\sum_{n=1}^{\infty} \frac{n}{n^3+1}$ lim. comp. with $\frac{1}{n^2}$ $\quad \lim \frac{\frac{n}{n^3+1}}{n^2} = \lim \frac{n^3}{n^3(1+1/n^3)} = \frac{1}{1} = 1$

& $\sum_{n=1}^{\infty} \frac{1}{n^2}$ is convergent by p-test

$\Rightarrow \sum_{n=1}^{\infty} \frac{n}{n^3+1}$ is convergent $\Rightarrow \sum_{n=1}^{\infty} (-1)^n \frac{n}{n^3+1}$ is absolutely convergent.

5. (6 pts.) Find the radius and the interval of convergence of $\sum_{n=0}^{\infty}(-1)^n \frac{(x+3)^n}{2n^2+1}$.

$$\lim_{n\to\infty} \left| \frac{(-1)^{n+1}(x+3)^{n+1}}{2n^2+4n+3} \cdot \frac{2n^2+1}{(-1)^n (x+3)^n} \right| = \lim_{n\to\infty} \left(|x+3| \frac{2n^2(1+1/2n^2)}{2n^2(1+1/2n+3/2n^2)} \right) = |x+3| < 1$$

$\Rightarrow$ Center $= -3$, Radius $= 1$

At $x = -4$: $\sum \frac{(-1)^n(-1)^n}{2n^2+1} = \sum_{n=1}^{\infty} \frac{1}{2n^2+1}$ (Conv. lim comp with $1/n^2$)

At $x = -2$, $\sum \frac{(-1)^n}{2n^2+1}$ is absolutely convergent (same as at $x=-4$)

$\Rightarrow$ Interval of convergence $[-4, -2]$ where the series is absolutely convergent even at the endpoints.